The Crisis of Expertise*

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Abstract

Decision makers receive expert advice on their actions to match a hidden, randomly evolving state. They are unsure whether the expert is actually informed. I find that a "crisis of expertise" can emerge, in which these decision makers dismiss an informed expert's correct advice and relies only on public information. Remarkably, this crisis happens precisely when public information has mediocre quality, and thus when the informed expert's knowledge is much needed. In contrast, high-quality public information preempts this crisis. I discuss policy implications of my analysis for alleviating this crisis.

Keywords: reputational cheap talk, public information.

JEL codes: C72, C73, D83.

1 Introduction

It is often argued that technology leads to a crisis of expertise by making information abundant and publicly accessible. While no unified definition of this crisis exists, one common description is that decision makers dismiss the genuine advice that informed experts offer and act solely on the basis of public information, for instance, from social

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media, because they doubt whether or not these experts are informed.[1] In this paper, I formalize this description of the crisis to address the following questions. When and why does the crisis of expertise arise? Is this crisis simply a phenomenon where decision makers substitute high-quality public information for expertise? How does this crisis affect the value of informed experts?

My main insight is that high-quality public information sustains decision makers' trust in expertise and their efficient use of informed experts' knowledge. The crisis of expertise happens precisely when the quality of public information is mediocre, and thus when informed experts' knowledge is much needed.

1.1 Overview of model and main result

In my model, a sequence of decision makers (DMs) pay for expert advice. Each DM takes an action to match a binary, randomly evolving hidden state given two pieces of information—current expert advice and public information—the latter of which consists of past advice and past state realizations. Only the expert knows whether she is an informed type who knows the state or an uninformed type who, like the DMs, does not know the state.

In formalizing the above description of the crisis of expertise, I focus on equilibria in which, on path, when giving advice to each DM, the informed type truthfully reports her private information that is relevant to the DM's action, namely her type and the current state. I also assume that the DMs' wage payments to the expert are competitive à la Holmström (1999), reflecting their perceived value of expert advice. This equilibrium focus and the competitive wages enable the prediction that during crises, the DMs dismiss the informed type's correct advice and their value of expert advice is zero.

In equilibrium, both expert types claim to be informed; thus, each piece of advice is uninformative about the expert's type and is plainly a state report. Before a DM receives a current report, he forms noisy public beliefs about both the state and the expert's type by using the public information. If he receives a report of the state that the public belief deems likely, then he matches his action with the report. This is because the report reinforces the public information. In contrast, he faces a trade-off upon receiving a report of the state that the public belief deems unlikely: he should

[1]For general discussions of this crisis, see Nichols (2017), Gurri (2018), and Eyal (2019).

match his action with this report if the expert is informed and should not do so otherwise. Thus, this DM matches his action with this report if and only if he believes that this report is sufficiently likely to be sent by an informed type.

A crisis of expertise is an equilibrium phenomenon where a DM matches his action with the likely state independently of an informed type's (correct) report. My main result derives a necessary and sufficient condition for this crisis to happen when the uninformed type is patient, i.e., when her payoff discounting is low. This condition is that the quality of public information about the state, as measured by the precision of the public state belief, is neither too high nor too low, just mediocre.

The intuition is as follows. If the public state belief is very precise, then the DM thinks that a report of the unlikely state is no likely to be sent by the uninformed type. This is because the uninformed type would find this report too likely to mismatch the state and in turn, reveal her type to future DMs and lead to zero future wages. Thus, the crisis does not happen. If the public state belief is very noisy instead, then the DM virtually does not value the public information about the state and uses the expert's report to guide his action. Thus, the crisis also does not happen. Conversely, if the public state belief has mediocre precision, then the DM thinks that the uninformed type is likely to report the unlikely state, because she gambles for higher future wages: in the unlikely event that this report turns out to match the state, her reputation, namely the public belief that she is informed, would rise, boosting her future wages. As a result, a crisis of expertise happens.

This main result contrasts with a typical view by economists that public information crowds out socially valuable private information.[2] In my model, public information does not crowd out the informed type's knowledge but may crowd out the DMs' trust in expertise. This latter event arises because public information has two faces: it not only informs decision-making, but also guides the uninformed type to gamble at the expense of the DMs.

1.2 Related literature

This paper belongs to the literature of reputational cheap talk (see, e.g., Scharfstein and Stein, 1990; Ottaviani and Sørensen, 2006a,b). Existing models ask how experts send cheap-talk messages to appear informed and show that these experts could exhibit

[2] See, e.g., Banerjee (1992) and Bikhchandani, Hirshleifer and Welch (1992) in the context of social learning and Morris and Shin (2002) in games with strategic complementarity.

misreporting behavior, including gambling (Trueman, 1994; Levy, 2004). These models abstract from decision-making, which is central to the crisis of expertise and key to my analysis. This distinction, in turn, motivates my methodological contribution, namely a microfoundation of the expert's reputational payoff based on competitive wages. Existing models typically take this payoff as an exogenously given, strictly increasing function of the expert's reputation, capturing the unmodeled utilities that the expert derives from her reputation. To make predictions about decision-making, however, more details about the structure of this reputational payoff are needed. This is because this structure shapes the uninformed type's gambling behavior, to which the DMs best reply. Imposing exogenous assumptions on this structure is unsatisfactory, for it is unclear what assumptions are appropriate. In my model, the reputational payoff and its structure arise endogenously as equilibrium phenomena.

Existing models in this literature are mostly static. As discussed in a survey by Marinovic, Ottaviani and Sørensen (2013), constructing a tractable dynamic model that captures behavior and learning about the expert's type and the state is challenging. My analysis takes a step towards this direction and obtains richer predictions than existing models do. My analysis shows that the crisis of expertise is a short-run phenomenon; in the long run, the informed type's wages reflect the true value of her advice. I also relate my results to public information policies, anecdotal and empirical evidence on expert advisors' behavior over their careers, as well as social conventions that reward successful contrarians and promote gambling behavior.

From a modeling perspective, Klein and Mylovanov (2017) is related. They study a finitely repeated game of reputational cheap talk where, as in my model, the expert receives competitive wages and her continuation payoffs associated with different reputations arise endogenously. Different from my analysis, they do not address the structure of these payoffs, as they do not characterize decision-making; rather, they focus on a patient expert's incentives to truthfully report her private information. Their expert does not know her own informativeness and does not gamble. Smirnov and Starkov (2019) and Shahanaghi (2022) also examine the dynamics of reputational cheap talk. Different from my analysis, they study how experts choose the timing of reports to appear informed; their experts can report only once and their reports are assumed to be truthful.

My assumption that the states are public realized after actions are taken is economically substantive, as it drives reporting incentives. This assumption is standard in the literature; it can be interpreted either literally, or indirectly by supposing that the DMs learn the realizations from an evaluator of the expert, such as a labor market. This latter interpretation is natural in my model, as the evaluator might represent a panel of referees who are established informed experts. To highlight the limitation of my results, as my analysis proceeds, I show that if the states remain hidden with high probabilities, then equilibrium predictions differ fundamentally. Thus, as in most existing models, my results apply to settings where information about past states is sufficiently accessible, but not otherwise. Some recent work explores settings where the states remain hidden and inferences about the expert's type can only be performed via indicators that are subject to the expert's manipulation (see, e.g., Rüdiger and Vigier, 2019; Camara and Dupuis, 2021).

To focus on the DMs' mistrust in expert informativeness, my model also follows the literature in assuming that the expert is impartial about the DMs' actions, contrary to models of partisan cheap talk (Crawford and Sobel, 1982), and that the expert takes no payoff-relevant actions, contrary to models of career concerns (e.g., Prendergast and Stole, 1996; Holmström, 1999).[3]

In my model, the crisis arises only because the uninformed type gambles to pool with the informed type who reports the true states on path. Thus, it is not a bad-reputation phenomenon where good experts' desire to separate from bad experts cause them to act against the DMs' interests; this latter phenomenon is captured in Morris (2001), where experts send cheap-talk messages, and in Ely and Välimäki (2003), where experts take payoff-relevant actions.[4] The uninformed type's pooling motive in my model is reminiscent of pooling in good reputation models à la Fudenberg and Levine (1989). Different from these models, the uninformed type's pooling is imperfect in my model. This observation, as I will show, is key to deriving my results.

[3]Thus, my model is different from dynamic partisan cheap talk models, starting with Sobel (1985) and Benabou and Laroque (1992). Recent work in this literature studies how dynamics affects communication efficacy despite players' action biases (e.g., Renault, Solan and Vieille, 2013; Margaria and Smolin, 2018; Meng, 2021; Best and Quigley, 2022; Kuvalekar, Lipnowski and Ramos, 2022; Mathevet, Pearce and Stacchetti, 2022; Pei, 2022).

[4]Gambling to pool with "good" types is not limited to the context of reputational cheap talk. For example, Lee and Liu (2013) explore this motive in a repeated bargaining game with different fundamentals and predictions. In their model, gambling entails short-run payoff consequences: a player gambles by rejecting attractive offers and taking her outside options so as to build a reputation for having good outside options; in equilibrium, a patient player gambles if and only if her reputation is neither too high nor too low. In my model, gambling entails no short-run consequences and in equilibrium, a patient uninformed type's gambling incentive is monotone: she gambles less given a higher reputation.

2 Example

I begin with an example that illustrates my main result and motivates my contribution to the literature. For this latter purpose, I cast this example in a standard, static framework of reputational cheap talk. Specifically, consider a decision maker (DM) who must pick an action, 0 or 1. He prefers action s in state $s \in \{0, 1\}$. For example, action 1 represents investing in a project and action 0 represents not doing so; this investment is profitable if and only if the state is 1. The DM does not know the state and so seeks expert advice before picking his action.

The expert privately knows whether she is an informed type or an uninformed type. This type is drawn independently of the state. She is informed with probability $p \in (0, 1)$ and is uninformed otherwise. The informed type knows the state. The uninformed type and the DM are equally ignorant about the state. They believe that $s = 0$ with probability $\mu \in [\frac{1}{2}, 1)$ and $s = 1$ otherwise; thus, they believe that the true state is (without loss) more likely to be 0. I call state 0 the likely state and call state 1 the unlikely state. I also call μ the public state belief. The interpretation is that the DM and the uninformed type form this belief based on all relevant public information about the state; a higher μ represents higher-quality public information.

The timing is as follows. The expert first learns her type. If she is informed, then she also learns the state. Next, she sends the DM a state report $m \in \{0, 1\}$. The DM then takes an action. Finally, the state is publicly realized. The DM's payoff is one if his action matches the state and is zero otherwise. The expert's payoff equals the DM's posterior belief $\hat{p}$ that she is informed, namely her reputation, once the state is publicly realized.

As motivated in the Introduction, I focus on perfect Bayesian equilibria in which the informed type reports the true state. The uninformed type's strategy is to pick a probability $\alpha \in [0, 1]$ to report 0 (and thus a probability $1 - \alpha$ to report 1). I say that a report is correct if it matches the state and is incorrect otherwise. The DM's strategy is to pick an action given his received report. I assume that he picks action 0 if he is indifferent between the two actions; this is without loss as his action affects neither expert type's payoff.

Finally, I assume that upon receiving a report that the DM expects from neither type, in which case Bayes' rule does not apply, the expert's (off-path) reputation is zero. I show in Appendix A.1 that this assumption is innocuous. Thus, given a report

m, a state realization s, and the DM's conjecture $\hat{\alpha}$ of the uninformed type's strategy, the posterior belief $\hat{p}$ is given by

$$\hat{p} = \mathbf{1}_{\{m=s=0\}} \left(\frac{p}{p + (1-p)\hat{\alpha}} \right) + \mathbf{1}_{\{m=s=1\}} \left(\frac{p}{p + (1-p)(1-\hat{\alpha})} \right), \tag{1}$$

where $\mathbf{1}_{\{\cdot\}}$ is the indicator function. Note that this posterior belief is positive if and only if the expert's report is correct.

2.1 Equilibrium

I begin by characterizing the equilibrium in Proposition 1 below. Then, in Proposition 2, I characterize the crisis of expertise as an equilibrium phenomenon. All proofs are in the Appendix. To state the results, it is essential to define the following cutoff

$$\kappa_{p,\mu} := \frac{\mu(2-p) - 1}{(2\mu - 1)(1-p)}. \tag{2}$$

Proposition 1. *There is a unique equilibrium in which the informed type reports the state truthfully. In this equilibrium:*

1. *The uninformed type reports state* 0 *with probability*

$$\alpha^*_{p,\mu} = \begin{cases} 1, & \text{if } \mu \geq \dfrac{1}{1+p}, \\ \mu \left(\dfrac{1+p}{1-p} \right) - \dfrac{p}{1-p}, & \text{otherwise,} \end{cases} \tag{3}$$

   and reports state 1 *with complementary probability.*

2. *If $\alpha^*_{p,\mu} \leq \kappa_{p,\mu}$, then the DM chooses action* 0 *irrespective of his received report. Otherwise, he matches his action with the report.*

Given the DM's equilibrium conjecture that the informed type reports the true state and the uninformed type plays $\alpha^*_{p,\mu}$, the informed type's best reply is indeed to report the true state, as an incorrect report leads to zero reputation; the uninformed type's strategy $\alpha^*_{p,\mu}$ is her best reply if

$$\alpha^*_{p,\mu} \in \arg\max_{\alpha \in [0,1]} \alpha\mu \frac{p}{p + (1-p)\alpha^*_{p,\mu}} + (1-\alpha)(1-\mu) \frac{p}{p + (1-p)(1-\alpha^*_{p,\mu})}. \tag{4}$$

Condition 4 holds because report 0 is correct with probability μ and report 1 is correct with probability $1-\mu$, and the posterior reputations are computed based on the DM's conjecture that the uninformed type plays $\alpha^*_{p,\mu}$.

The uninformed type's strategy (3) reflects her two desires. First, she wishes to choose a high α in (4), as report 0 is more likely to be correct. However, if the DM's conjecture $\alpha^*_{p,\mu}$ is too high, then the DM believes that report 1 is likely to be sent by the informed type. Then, the expert's reputation upon a correct report 1 is much higher than that upon a correct report 0. The uninformed type then deviates to only report 1, gambling on the unlikely event that report 1 is correct for a high reputation. In equilibrium, she gambles with positive probability unless the quality of public information μ is high enough, in which case report 1 is very unlikely to be correct.

The DM's incentive is as follows. Given report 0, he optimally matches his action with the report, as this report reinforces the public information. In contrast, given report 1, the DM faces a trade-off: he should take action 0 despite receiving this report if the expert is uninformed, but should match his action with this report otherwise. In equilibrium, the DM takes action 0 despite receiving report 1 if and only if the uninformed type is likely to gamble by reporting 1, i.e., if and only if $\alpha^*_{p,\mu} \leq \kappa_{p,\mu}$.

2.2 The crisis of expertise

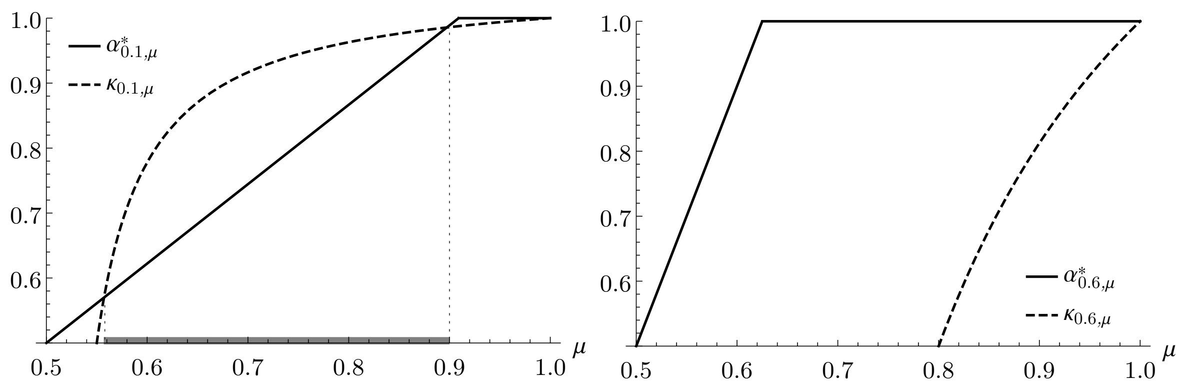
In this equilibrium, I say that a crisis of expertise arises if the DM's strategy is to take action 0—the action that matches the likely state—independently of the expert's report, conditional on the expert being an informed type.

By Proposition 1, a crisis arises if and only if $\alpha^*_{p,\mu} \leq \kappa_{p,\mu}$. Proposition 2 below characterizes this crisis in terms of the primitives.

Proposition 2. *In the equilibrium, there exist* $\underline{\mu}^* \equiv \underline{\mu}^*(p)$, $\bar{\mu}^* \equiv \bar{\mu}^*(p)$, *with* $\frac{1}{2} < \underline{\mu}^* \leq \bar{\mu}^* < 1$, *such that:*

1. *A crisis of expertise happens if and only if* $\mu \in [\underline{\mu}^*, \bar{\mu}^*]$.
2. *There exists* $p^* \in (0,1)$ *such that:*
   (a) $[\underline{\mu}^*(p), \bar{\mu}^*(p)]$ *is nonempty if and only if* $p \leq p^*$;
   (b) *For any* p, p' *that satisfy* $0 < p < p' \leq p^*$,

$$[\underline{\mu}^*(p'), \bar{\mu}^*(p')] \subsetneq [\underline{\mu}^*(p), \bar{\mu}^*(p)].$$



(a) $p = 0.1$ **(b)** $p = 0.6$

Figure 1: The crisis of expertise (gray area: the crisis region $[\underline{\mu}^*, \bar{\mu}^*]$)

Part 1 characterizes when the crisis arises and when it does not. It follows from simplifying (2) with (3). If $\mu > \bar{\mu}^*$ so that high-quality public information undermines the uninformed type's gambling incentive, then the crisis does not arise. If $\mu < \underline{\mu}^*$ so that public information has low quality, then the DM uses the expert's report to guide his action and the crisis also does not arise. Conversely, if $\mu \in [\underline{\mu}^*, \bar{\mu}^*]$, then the DM views public information as fairly accurate and the uninformed type's gambling incentive as fairly strong. The crisis thus arises.

Part 2 states that this crisis region $[\underline{\mu}, \bar{\mu}]$ shrinks as the expert's reputation p increases and is empty for high enough reputations. This happens for two reasons. Given a higher reputation p, first, the cutoff $\kappa_{p,\mu}$ given in (2) is lower, capturing the DM's belief that the report is more likely to be sent by the informed type; second, $\alpha^*_{p,\mu}$ is higher, capturing the uninformed type's weaker gambling incentive as the reputation gain upon a correct report 1 is then smaller. Figure 1 illustrates this proposition.

2.3 Implications and limitations

Proposition 2 addresses two of my opening motivating questions. First, the crisis of expertise is *not* a phenomenon where the DM substitutes high-quality public information for expert advice. In contrast, high-quality public information sustains the DM's trust in expert advice by disciplining the uninformed type's gambling. The DM dismisses expert advice if and only if the quality of public information is mediocre.

This example, nonetheless, does not address my last motivating question concerning the value of the informed type. Moreover, importantly, Proposition 2 relies on the assumption that the uninformed type's payoff is linear in her reputation, given which her equilibrium strategy $\alpha^*_{p,\mu}$ is piecewise linear in μ. This piecewise linearity, in turn, enables the sharp characterization given in Proposition 2, as Figure 1 has illustrated. This paper establishes this characterization more generally. To this end, one must ask what structure the uninformed type's reputational payoff should exhibit. For the above reasons, I next turn to my main analysis, in which the uninformed type's reputational payoff and its structure, as well as the value of the informed type's advice, arise endogenously as equilibrium phenomena.

3 Model

Time $t = 0, 1, \dots$ is discrete and the horizon is infinite. A long-lived expert faces a sequence of short-lived decision makers (DMs). The expert has a type $\theta \in \{\theta_I, \theta_U\}$: she is an informed type θ_I with probability $p_0 \in (0, 1)$ and an uninformed type θ_U otherwise.

Timing, actions, and payoffs. In each period t, a state $s_t \in S := \{0, 1\}$ and a quality $\mu_t \in [\frac{1}{2}, 1)$ of public information about this state are drawn, independently across time and independently of the expert's type. Specifically, the quality μ_t is first drawn according to a distribution F with support $[\frac{1}{2}, 1)$. Then, the state s_t is drawn to be 0 with probability μ_t and be 1 otherwise. Thus, the true state is (without loss) more likely to be 0 *ex ante*. Throughout, I call state 0 the likely state and call state 1 the unlikely state.

Then, a DM arrives and pays the expert a wage $w_t \in \mathbf{R}_+$ that I specify below in (5). Next, the expert sends the DM a message $m_t \in M$, where M is a finite set. Finally, the DM takes an action $a_t \in S$. In this period, the DM obtains a benefit $b_t = 1$ if his action matches s_t and a benefit $b_t = 0$ otherwise. The DM's payoff is his benefit minus the wage, i.e., $b_t - w_t$. The expert's payoff is her wage w_t.

The expert has a discount factor $\delta \in (0, 1)$. Thus, in the repeated game, the expert's normalized, realized payoff is $(1 - \delta) \sum_{t=0}^{\infty} \delta^t w_t$.

Information. The expert privately knows her type. In each period t, the quality μ_t is publicly observed. In contrast, the state s_t is hidden until the DM picks his action, at which point this state is publicly realized. The message m_t and the action a_t are publicly observed.[5] Before sending a message, the expert receives a private signal $y_t \in S$. If she is an informed type, then this signal equals the state s_t. If she is an uninformed type, then this signal is either 0 or 1 equiprobably and is pure noise. Thus, the uninformed type and the DMs are equally ignorant about the state.

Histories, strategies, and beliefs. A public history in period t is a list $h_t = (\mu_0, m_0, a_0, s_0, \ldots, \mu_{t-1}, m_{t-1}, a_{t-1}, s_{t-1}, \mu_t)$, collecting the past public information qualities, messages, actions, and state realizations, as well as the current public information quality. A type-θ expert's private history in period t is a list $h_t^\theta = (h_t, y_0, \ldots, y_{t-1})$, consisting of the public history and her past signals. Let H_t be the set of period-t public histories. Let H_t^θ be the set of type-θ expert's period-t private histories.

Each type-θ expert's strategy is a collection $\sigma^\theta = (\sigma_t^\theta)_{t=0}^\infty$, where each $\sigma_t^\theta : H_t^\theta \times S \to \Delta M$ maps her history h_t^θ and her current signal y_t to a distribution from which her period-t message is drawn. Each period-t DM's strategy is to take an action given the public history h_t and the current message m_t. If a DM is indifferent between the two actions, I assume that he takes action 0; this is without loss, as his action does not affect the information of the expert, the information of future DMs, and future wages.

Finally, given a public history h_t in each period t, the DM's state belief (that the current state is 0) is given by μ_t; he also forms a belief p_t that the expert is informed. This belief p_t captures the expert's reputation. In any period, I refer to a generic pair (p, μ), consisting of the expert's reputation and the DM's state belief as the public beliefs.

Wages. I follow the literature on reputations (e.g., Holmström, 1999) in assuming that the expert receives competitive compensations. Specifically, in each period t, the DM's wage payment w_t is equal to his perceived value of receiving a message from the expert:

$$w_t := \hat{\mathbf{E}}\left[b_t^*\left(h_t, m\right) | h_t\right] - \mu_t, \tag{5}$$

where $b_t^*(h_t, m_t)$ denotes the DM's expected benefit from optimal choosing an action upon receiving current message m_t given public history h_t, $\hat{\mathbf{E}}[\cdot|h_t]$ denotes the expectation taken over the expert's current messages m_t according to the DM's conjecture

[5] My results are unchanged if the DM's action is his private information.

of both expert types' strategies given the public history h_t, and finally, μ_t captures the DM's expected optimal benefit from taking an action without receiving a current message from the expert, in which case he would optimally take action 0.[6]

Equilibrium. The solution concept that I use is perfect Bayesian equilibrium. In the spirit of Myerson (1986), I assume that in equilibrium, the set of messages is $M = \{\theta_U, \theta_I\} \times S$, allowing each type to disclose all her private information that is relevant to the DM's action in each period given the public history, namely her type and her current signal.

This game has many equilibria, including those in which the informed type's messages are uninformative about the states. As motivated in the Introduction, I focus on equilibria satisfying Properties 1—3 below. Throughout, I refer to equilibria satisfying these three properties as *reputational equilibria.*

Property 1. *On path, in each period, the informed type reports her private information, namely her type and her current private signal, truthfully.*

Property 1 rules out equilibria in which the informed type babbles or mixes between truthfully reporting and misreporting her private information. It aligns with my objective to formalize the crisis of expertise as a phenomenon where a DM dismisses the informed type's correct advice.

Given this property, I assume without loss that the uninformed type never reports her true type, as this report leads to zero future wages. Accordingly, I simplify the structure of the message set and assume throughout that $M = S$. A message is interpreted as a report of the current state. I say that a report is correct if it matches the current state, and is incorrect otherwise. Property 1 requires that in equilibrium, the informed type reports correctly on path. Note that in such an equilibrium, she has no profitable deviation to misreport on path, as this deviation leads to zero future wages. On the other hand, without loss, I take the uninformed type's strategy to be a probability of reporting the likely state, i.e., state 0, in each period given her history (and her uninformative current signal).

[6] To see how $b_t^*(h_t, m_t)$ is defined in terms of primitives, let $\beta(a_t, s_t)$ denote the DM's realized benefit in period t if he chooses action a_t and the state is s_t. Then, $b_t^*(h_t, m_t) := \max_{a \in S} \hat{\mathbf{E}}[\beta(a, s_t)|h_t, m_t]$, where the expectation $\hat{\mathbf{E}}$ is taken over the set of states, conditional on the public history and the expert's current message.

Property 2 ensures that both types benefit from a higher reputation. Given an equilibrium, let $v^\theta(p; h^\theta)$ denote the type-θ expert's payoff at her history h^θ with associated reputation p.

Property 2. *For each type θ, given any history h^θ on path, $v^\theta(p; h^\theta)$ is strictly increasing in p.*

Contrary to standard models of reputational cheap talk, this continuation payoff v^U and Property 2 must arise endogenously in equilibrium.[7]

Finally, because Bayes' rule does not apply when beliefs are degenerate, Property 3 below, which is commonly imposed in the literature (see, e.g., Rubinstein, 1985), identifies continuations with degenerate reputations as complete-information games.[8]

Property 3. *Degenerate reputations are absorbing: if at any history of play, the reputation is $p = 0$ (resp., $p = 1$), then at any concatenation of the history, the reputation is also 0 (resp., 1).*[9] *In addition, in each period with reputation $p = 0$, the DM's state belief is μ irrespective of the report he receives.*

Discussion of modeling assumptions. As described in the Introduction, my assumption that the state is publicly realized at the end of each period is economically substantive. In Section 6, I concretely illustrate this issue. In that section, I also discuss some extensions of my analysis.

I next describe some innocuous assumptions that I have made. The restriction to two states plainly simplifies the exposition. This is because each DM's best-reply problem is effectively to decide whether to match his action with his report against choosing his best alternative, namely action 0, but not any other action even if there are more than two states. Moreover, by assumption, the expert reports even when her wage is zero. That is, my model abstracts from the DM's decision to hire the expert

[7]Similar monotonicity conditions are imposed in models outside of the literature of reputational cheap talk. See, e.g., Fudenberg, Levine and Tirole (1987) and Lee and Liu (2013) in the context of bargaining, and Benabou and Laroque (1992), Mathis, McAndrews and Rochet (2009) in the context of pooling reputations.

[8]To be sure, this property is not void of problems. Madrigal, Tan and da Costa Werlang (1987) illustrate how this property might conflict with equilibrium existence; see Nöldeke and van Damme (1990) for further discussion concerning its shortcomings.

[9]More precisely, a history of play at time t, denoted by η_t, consists of the type, the public history, and the expert's past signals at time t. A concatenation of the history of play η_t followed by another history of play $\eta_{t'}$ is a history of play, denoted by $\eta_t\eta_{t'}$, at time $t + t'$. A concatenation of η_t is a history of play $\eta_t\eta_{t'}$ for some $\eta_{t'}$.

or not. My results are unchanged if the expert does not send a message when the DM does not value receiving her advice, although the DMs' learning of the expert's type would then be slower. Finally, the mechanism that drives my results is unaffected if the two expert types share different discount factors, if the DMs' benefits are asymmetric across the states, and if these benefits are not normalized to zero and one.

4 The crisis of expertise

In this section, I define a crisis of expertise and characterize it.

Definition 1. *In any reputational equilibrium, a crisis of expertise is a history of play on path at which a DM chooses action* 0 *irrespective of the expert's current report, conditional on the expert being informed.*

My main result is:

Proposition 3. *There exists* $\underline{\delta} \in (0,1)$ *such that for every* $\delta \geq \underline{\delta}$*, a reputational equilibrium exists. In each reputational equilibrium, there exist* $\underline{\mu} \equiv \underline{\mu}(p)$ *and* $\bar{\mu} \equiv \bar{\mu}(p)$*, with* $\frac{1}{2} < \underline{\mu} \leq \bar{\mu} < 1$ *so that in each period with public beliefs* $(p, \mu) \in [p_0, 1] \times [\frac{1}{2}, 1)$*:*

1. *A crisis of expertise happens if and only if* $\mu \in [\underline{\mu}, \bar{\mu}]$*.*
2. *There exists* $\bar{p} \in (0,1)$ *such that:*
   (a) $[\underline{\mu}(p), \bar{\mu}(p)]$ *is nonempty if and only if* $p \leq \bar{p}$*;*
   (b) *For any* p, p' *that satisfy* $0 < p < p' \leq \bar{p}$*,*

$$[\underline{\mu}(p'), \bar{\mu}(p')] \subsetneq [\underline{\mu}(p), \bar{\mu}(p)].$$

Proposition 3 gives a necessary and sufficient condition for the crisis of expertise to happen in each period when the expert is sufficiently patient. This proposition mirrors Proposition 2, with the new requirement of low discounting. Observe that Proposition 3 does not concern periods in which the expert's reputation falls short of the prior reputation p_0. The reason is that in any reputational equilibrium, the crisis of expertise is a phenomenon on path conditional on an informed type; in all periods on path, the informed type reports correctly and her reputation is at least p_0.

The intuition behind Proposition 3 is identical that of Proposition 2. I use the remainder of this section to sketch the proof of Proposition 3, and then discuss the role of low discounting. In particular, as I will explain, this result indeed requires only the uninformed type's discounting to be low.

4.1 Equilibrium structure

The crisis of expertise, by definition, is an equilibrium phenomenon. Lemma 1 below begins by examining the structure that any reputational equilibrium must exhibit.

Lemma 1. *In any reputational equilibrium, the following holds in each period.*

1. *If there is at least one incorrect past report, then:*
   - (a) *It is without loss to assume that both types report the two states equiprobably. Moreover, their continuation payoffs are zero.*
   - (b) *The DM chooses action* 0 *regardless of the expert's report.*
2. *If there is no incorrect past report and the public beliefs are* (p, μ)*:*
   - (a) *The uninformed type's strategy is characterized by* (p, μ)*: she reports* 0 *with some probability* $\alpha_{p,\mu} \in [\frac{1}{2}, 1]$*. Once the state is publicly realized, if her current report turns out to be correct, then her reputation is updated to some* $p' \geq p$ *and her continuation payoff is* $V^U(p')$*, where* $V^U : [p_0, 1] \to \mathbf{R}$ *is a continuous and strictly increasing function.*
   - (b) *The informed type's strategy is to report the current state. Once the state is publicly realized, her reputation is updated to some* $p' \geq p$ *and her continuation payoff is* $V^I(p')$*, where* $V^I : [p_0, 1] \to \mathbf{R}$ *is a continuous and strictly increasing function.*
   - (c) *The DM's strategy is as follows. If* $\alpha_{p,\mu} \leq \kappa_{p,\mu}$*, where* $\alpha_{p,\mu}$ *is given above and* $\kappa_{p,\mu}$ *is given in* (2)*, then the DM chooses action* 0 *irrespective of the expert's report; otherwise, he matches his action with the report.*

Part 1(a) says that both types' equilibrium payoffs are zero in each period with at least one incorrect past report. By Property 1, this period only happens off path for the informed type. Part 1(a) is plainly a consequence of Bayes' rule if an expert reported incorrectly when her reputation is short of one, as the DMs then infer that she is uninformed and pay her zero wages. It is, however, not a consequence of Bayes' rule if the expert reported incorrectly when her reputation is one, in which case her reputation remains as one by Property 3.[10] Indeed, Lemma 1 shows that following this history, the informed type must stop sending reports that are valuable to the DMs, e.g., the informed type repeatedly sends reports independently of her signals, so that both types' wages and continuation payoffs are zero. Otherwise, the uninformed type's payoff is discontinuous at reputation one, precluding equilibrium existence.

Thus, in this period, it is without loss to assume that both types report both states equiprobably, because their reports do not affect their payoffs. In turn, as Part 1(b) states, the DM's optimal strategy in this period is to take action 0, regardless of his received report.

Part 2(a) concerns the uninformed type's equilibrium payoff and strategy in each period with no incorrect past report. The uninformed type's payoff V^U is a strictly increasing function of her reputation, as she benefits from a higher reputation in view of Property 2. Because the expert's reputation is at least the prior reputation p_0 if she has never reported incorrectly, V^U is defined on $[p_0, 1]$. In this period, the uninformed type's equilibrium strategy $\alpha_{p,\mu}$ is characterized by the beliefs (p, μ) because, when choosing what to report, she takes into account the chance that each report is correct, as determined by μ, and her current reputation p as it determines her future reputation. This strategy maximizes her payoff when the DMs conjecture that she chooses this strategy (and the informed type reports the true state):

$$\begin{aligned}\alpha_{p,\mu} \in \arg\max_{\tilde{\alpha}\in[0,1]} \ & \mu\tilde{\alpha}V^U\left(\frac{p}{p+(1-p)\alpha_{p,\mu}}\right) \\ & + (1-\mu)(1-\tilde{\alpha})V^U\left(\frac{p}{p+(1-p)(1-\alpha_{p,\mu})}\right).\end{aligned} \tag{6}$$

As in Section 2, $\alpha_{p,\mu}$ balances the two desires of the uninformed type, namely to report 0 to best match the state as well as to report 1 to gamble for her reputation.

Next, Part 2(b) concerns the informed type's equilibrium payoff and strategy. It follows almost immediately from Properties 1 and 2. Her equilibrium payoff is a function of her reputation because her expected wage in each period without an incorrect report, prior to the quality of public information μ being drawn, is completely determined by her current reputation p. The reason is that given any realization μ, by (5), her wage is determined by the DM's conjecture of both types' strategies: the informed type reports correctly and the uninformed type's strategy is characterized

[10]Note that an uninformed type might earn a reputation of one off path.

by (p, μ).

Finally, Part 2(c) describes the DM's equilibrium strategy in this period. Because the DM is short-lived, his strategy is analogous to that in Proposition 1 in Section 2, with the same cutoff $\kappa_{p,\mu}$ given in (2).

4.2 Uninformed gambling

The crisis of expertise, by definition, is a equilibrium phenomenon that arises conditional on an informed type, who never reports incorrectly on path. Thus, the crisis happens only in periods with no incorrect past report. In any such period, if the public beliefs are (p, μ), then in view of Part 2(c) of Lemma 1, a crisis of expertise happens if and only if the uninformed type's strategy $\alpha_{p,\mu}$ satisfies $\alpha_{p,\mu} \leq \kappa_{p,\mu}$ in view of Part 2(c) of Lemma 1.

Thus, to characterize the crisis, Lemma 2 below examines the equilibrium structure of the uninformed type's strategy in each period with no incorrect past report in detail. Hereafter, fix a reputational equilibrium. I shall refer to this equilibrium as "the reputational equilibrium." Let $\{\delta_n\}_{n=0}^{\infty}$ be an increasing sequence of discount factors converging to one. Let α^n be the uninformed type's report function that determines the uninformed type's strategy $\alpha_{p,\mu}$ in a period with no incorrect past report and with public beliefs (p, μ) in the reputational equilibrium when the discount factor is δ_n. Finally, Let $\alpha^* : [p_0, 1] \times [\frac{1}{2}, 1) \to [0, 1]$ be a report function that determines $\alpha^*_{p,\mu}$ in (3) given beliefs (p, μ) in Section 2.

Lemma 2. *The following holds.*

1. *Fix n. For each $p \in [p_0, 1]$, there exists $\mu^n_\dagger \equiv \mu^n_\dagger(p) \in (\frac{1}{2}, 1)$ such that*
   (a) *$\alpha^n_{p,\mu} < 1$ and it is strictly increasing in μ on $[\frac{1}{2}, \mu^n_\dagger)$;*
   (b) *$\alpha^n_{p,\mu} = 1$ for $\mu \in [\mu^n_\dagger, 1)$.*

2. *For every $\varepsilon > 0$, there exists $N > 0$ such that for every $n \geq N$:*
   (a) *Uniform convergence of report function:*

$$\sup_{(p,\mu)\in[p_0,1]\times[\frac{1}{2},1)} \left|\alpha^*_{p,\mu} - \alpha^n_{p,\mu}\right| < \varepsilon. \tag{7}$$

(b) Convergence of curvature of report function:

$$\sup_{(p,\mu,\mu')\in[p_0,1]\times[\frac{1}{2},1)^2:\mu\neq\mu'} \left| \frac{\alpha^*_{p,\mu'}-\alpha^*_{p,\mu}}{\mu'-\mu} - \frac{\alpha^n_{p,\mu'}-\alpha^n_{p,\mu}}{\mu'-\mu} \right| < \varepsilon. \tag{8}$$

(c) Monotonicity of gambling in reputation: $\alpha^n_{p,\mu}$ is increasing in p for each $\mu \in [\frac{1}{2}, 1)$.

Part 1 shows that the uninformed type's gambling incentives captured by the strategy $\alpha^n_{p,\mu}$ is similar to those captured by $\alpha^*_{p,\mu}$: she gambles for her reputation by reporting 1 with a higher probability when the quality of public information μ is lower, and she does not gamble at all if μ is high enough. The reason is that V^U is strictly increasing, so that the uninformed type's incentives in her best reply problem in (6) resemble those in her best reply problem (4) in Section 2.

Part 2 sharpens the prediction of the uninformed type's strategy $\alpha^n_{p,\mu}$ when discounting is low, i.e., when n is large; it shows that $\alpha^n_{p,\mu}$ "approximates" the strategy $\alpha^*_{p,\mu}$ given by (3) in Section 2. Intuitively, in each period, the uninformed type reports incorrectly with positive probability, upon which she collects zero wages thereafter. Thus, if (the uninformed type's) discounting vanishes, then the uninformed type's payoff V^U in the reputational equilibrium uniformly vanishes; with low discounting, V^U is approximately linear. In turn, the uninformed type's best reply problem (6) is virtually her best reply problem (4) in Section 2.

Given Part 2 of Lemma 2, by invoking Propositions 1 and 2, the characterization of the crisis in Parts 1 and 2 of Proposition 3 follows. It remains to show that a reputational equilibrium exists when discounting is low, as I next sketch.

4.3 Existence

When discounting is low, the strategy profile described in Lemma 1 constitutes a reputational equilibrium. By construction, it constitutes an equilibrium that satisfies Properties 1 and 3 irrespective of discounting. Finally, as a consequence of Lemma 3 below, this strategy profile also satisfies Property 2 when discounting is low.

Lemma 3. *Given the equilibrium strategy profile specified in Lemma 1, there exists $\underline{\delta} \in (0,1)$ such that for every $\delta \geq \underline{\delta}$, in each period where the expert's reputation is*

$p \in [p_0, 1]$, *her expected wage prior to the current public information quality* μ *being drawn is strictly increasing in* p.

To understand this result, consider a period with public beliefs $(p, \mu) \in [p_0, 1] \times [\frac{1}{2}, 1)$ given the equilibrium strategy profile specified in Lemma 1. By Part 1 of Proposition 3, if $\mu \in [\underline{\mu}, \bar{\mu}]$, then the DM chooses action 0 for sure, so that the expert's report is not valuable to him. The wage is then zero. If $\mu \notin [\underline{\mu}, \bar{\mu}]$ instead, then the DM matches his action with the expert's report, and his wage payment is positive. By (5), this wage is equal to

$$p + (1 - p)(\mu\alpha_{p,\mu} + (1 - \mu)(1 - \alpha_{p,\mu})) - \mu. \tag{9}$$

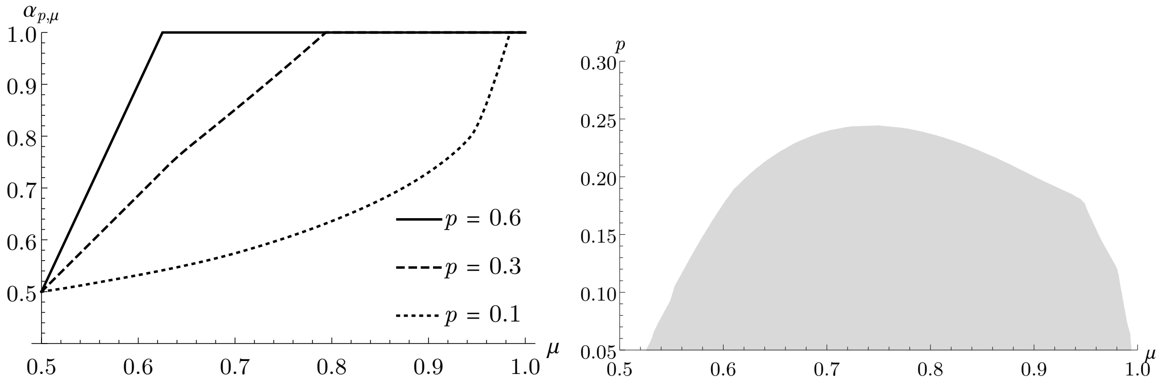
When discounting is low, by Part 2(c) of Lemma 2, the uninformed type is less likely to gamble given a higher reputation so that $\alpha_{p,\mu}$ is increasing in p. In turn, (9) is strictly increasing in p. Because there is a positive measure of qualities μ given which the expert's wage is positive, the expert's expected wage prior to μ being drawn is strictly increasing in p and is positive. This, in turn, implies that both types' continuation payoffs in any reputational equilibrium are strictly increasing in p.

4.4 The role of low discounting

Low discounting is key to Proposition 3 for two reasons. First, it ensures that the uninformed type's gambling incentive is monotone in her reputation, according to Part 3 of Lemma 2, which in turn ensures that a reputational equilibrium exists in view of Section 4.3. Second, it ensures a sharp prediction of the uninformed type's gambling behavior relative to the cutoff (2), and thus a sharp characterization of the crisis region as stated in the proposition.[11]

As is standard, low discounting here can be interpreted as a literal description that the uninformed type considers future wages as sufficiently important. Alternatively, it can be interpreted as the uninformed type's view that the game continues with a high probability δ and ends otherwise at the end of each period, so that she expects to be able to exploit her undeserving reputation for a long time until her type is exposed.

[11] To be sure, these two reasons differ from the usual appeal to low discounting in the literature on repeated games, which is to provide players with appropriate incentives by affecting their intertemporal trade-offs. Here, the uninformed type's trade-off when choosing her strategy in each period is not intertemporal, because she has already collected her current wage. Instead, her trade-off arises from comparing her future payoff upon report 0 with her future payoff upon report 1, as (6) makes clear.



(a) the uninformed type's strategy

(b) the crisis region $[\underline{\mu}(p), \bar{\mu}(p)]$

Figure 2: Numerical simulations ($p_0 = 0.05$, $\delta = 0.6$, $F = 1 \circ \{0.65\}$)

When discounting is not sufficiently low, I can neither prove nor disprove that a reputational equilibrium exists, let alone determining whether the characterization in Proposition 3 extends. Numerical simulation, nonetheless, is straightforward and suggests a similar characterization. Consider the following numerical example depicted in Figure 2. In this example, I set the discount factor to be $\delta = 0.6$, and assume that the distribution of public information quality F draws $\mu = 0.65$ with probability one in each period. The uninformed type's strategy $\alpha_{p,\mu}$ appears to be monotone in her reputation, so that the argument in Section 4.3 for both types' equilibrium payoffs to be monotone can be adopted. The curvature of this strategy varies considerably over the beliefs (p, μ). In turn, given each reputation p, a sharp prediction of the set of state beliefs μ given which $\alpha_{p,\mu} \leq \kappa_{p,\mu}$, so that a crisis emerges, is difficult to obtain. Numerically, as shown in Figure 2b, the crisis region appears to satisfy the characterization as given in Proposition 3.

5 Implications

I next turn to the implications of my results, addressing the motivating questions.

5.1 Public information and expert advice

My results suggest that the crisis of expertise is *not* a phenomenon where the DMs substitute high-quality public information for expert advice. Rather, high-quality public information complements the DMs' trust in expert advice and their efficient

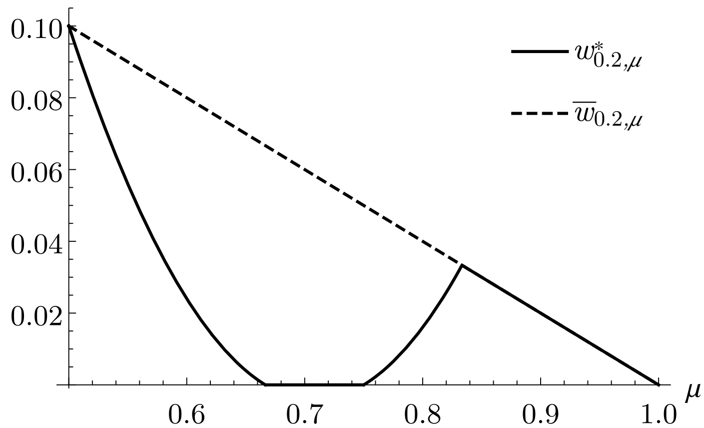


Figure 3: The value of expert advice and uninformed gambling

use of the informed type's knowledge. The DMs dismiss expert advice if and only if the quality of public information is mediocre.

My analysis has elucidated that public information about the state has two faces: it not only informs the DMs' actions, but also guides the uninformed type to gamble at the expense of the DMs. Wages reflect how these two faces of public information affect the DMs' value of informed advice: in view of the discussion following Lemma 3, this value is zero in a crisis and is positive, as given by (9), otherwise.

To illustrate these two faces, Figure 3 plots two different wages as a function of the quality of public information μ by fixing the expert's reputation at $p = 0.2$. The function $w^*_{p,\mu}$ corresponds to the expert's wage in a reputational equilibrium whose strategy profile is specified in Lemma 1 in the limit of no discounting. The function $\bar{w}_{p,\mu}$ is a benchmark wage when a DM (incorrectly) conjectures that the uninformed type never gambles in this equilibrium: by (5), $\bar{w}_{p,\mu} = p(1-\mu)$. This benchmark wage shows that public information unambiguously improves a DM's decision-making absent uninformed gambling. As the quality of public information improves, the DM's marginal benefit from receiving expert advice falls. The limiting equilibrium wage then highlights the detrimental impact of uninformed gambling on the value of expert advice. When the quality of public information is not high enough so that the uninformed type gambles with positive probability, the value of expert advice falls and in particular, it is destroyed completely when the quality of public information is mediocre, i.e., in a crisis of expertise.

In recent decades, political campaigns and consumer groups often unambiguously advocated the need to equip consumers with high-quality public information (see, e.g., Howells, 2005); the idea is to protect them from low-quality expert advice. My results support these advocates and also provide a new argument: policies improving the quality of public information not only protect consumers by improving their decision-making without expert advice, but are also effective measures for alleviating the crisis of expertise.

Relatedly, Proposition 3 also suggests that high standards of expert certifications, which ensure high expert reputations, can be effective in sustaining trust in expertise.

5.2 Gambling and contrarian-rewarding conventions

In my analysis, both the uninformed type's gambling behavior and the contrarian-rewarding convention—the large payoff reward upon a correct report 1 relative to a correct report 0—that sustains gambling behavior arise endogenously as equilibrium phenomena. These phenomena are widely documented, for instance, in financial markets and entrepreneurship (see, e.g., Baddeley, 2018; Bozanic, Chen and Jung, 2019).

Bozanic et al. (2019) also document that financial analysts who work at lower-tier brokerage houses are more likely to make calls contrary to the public beliefs for their career advancement. This aligns with the prediction in the reputational equilibrium that on average, the expert is more likely to report the unlikely state given a lower reputation: the informed type reports correctly irrespective of her reputation, and the uninformed type is more likely to gamble by reporting state 1 in view of Lemma 2.

5.3 The long-run value of informed advice

Finally, I examine the long-run value of informed type's advice, as measured by her wage in the reputational equilibrium. As discussed in Section 5.1, the (short-run) value of informed advice is zero in a crisis. In contrast, in any period where the quality of public information is given by μ, if the DM knew the informed expert's type, then the wage is equal to $1 - \mu$, according to (5).

Corollary 1 below shows that wages fully reflect the true value of the informed type's reports in the long run in the reputational equilibrium. The reason is that the DMs learn about the informed expert's type given her correct reports over time.

Corollary 1. *Let* $\mathbf{P}$ *denote the probability measure over the set of outcomes induced in the reputational equilibrium. Then, for every* $\varepsilon > 0$*, there exists* $T > 0$ *such that* *for every* $t \geq T$,

$$\mathbf{P}\left[w_t \geq 1 - \mu_t - \varepsilon | \theta = \theta_I\right] \geq 1 - \varepsilon. \tag{10}$$

This result sheds light on why, despite concerns about the crisis of expertise, reputable experts who have consistently issued correct forecasts are competitively sought after in financial markets (see, e.g., Hong, Kubik and Solomon, 2000; Hong and Kubik, 2003), business consulting (see, e.g., Bourgoin and Harvey, 2018), or entrepreneurship (see, e.g., Baddeley, 2018).[12]

Nonetheless, the informed type's realized wage path $(w_t)_{t=0}^{\infty}$ in the reputational equilibrium need not be increasing. This is because the quality of public information μ, which constitutes the DMs' "reservation payoffs," differs across periods. Conversely, if the quality μ is identical across periods, then $(w_t)_{t=0}^{\infty}$ is increasing in any reputational equilibrium when discounting is low. This is because the informed type's reputation is increasing over time on path and the DMs expect weaker gambling incentives from a more reputable uninformed type by Part 3 of Lemma 2:

Corollary 2. *If the distribution* F *that draws the quality* μ *is degenerate, then there exists* $\underline{\delta} \in (0,1)$ *such that for every* $\delta \geq \underline{\delta}$*, any realized sequence* $(w_t)_{t=0}^{\infty}$ *of the informed type's wages is increasing in the reputational equilibrium.*

5.4 A case in financial advice

Finally, I relate the above discussion to recent concerns in the financial advice industry. Take, for example, the case of Patricia Russell, who appeared in 2019 on *LinkedIn* as a certified financial planner and a graduate from prestigious universities. Her financial advice was quoted in major media outlets, allowing Patricia to benefit from an undeserving reputation as a financial expert before she was discovered to be fake.[13] The discovery of Russell and other fake financial advisors attracted considerable concerns that the presence of these fake experts diminishes the value of informed

[12]Of course, there could be factors outside of my model that disrupt this result. For example, the DMs might have no access to the track records of individual experts, and they perceive an expert's competence based on the collective reputation of the profession that the expert belongs to. If these DMs are sufficiently worried about the "bad apples" in the profession, then the collective reputation might never be high enough to sustain their trust in expert advice. Corollary 1 points to the importance of policies that make experts' individual track records publicly accessible.

[13]See "How This Fake Financial Expert Tricked Outlets Into Publishing Her Advice", *Huffpost*, August 1, 2019.

financial advisors and their genuine advice.[14] The two faces of public information, as discussed above, shed light on why these fake experts spring to life on the Internet.

My results predict that these fake experts tend to attract attention by initially making contrarian statements, and gradually tend to herd with public beliefs as their reputations rise until their identities are exposed. Although their presence damages consumers' trust in the informed advisors and thus the value of their genuine advice, this damage vanishes in the long run by Corollary 1, so long as that these informed experts' track records are accessible to consumers.

6 Discussion

In this section, I highlight the limitation of my results by discussing the role of the economically substantive assumption, namely that the state is publicly realized at the end of each period. I then comment briefly on how my analysis can be enriched. For simplicity, I cast the results in this section by using the example in Section 2, and adopt the definition of a crisis of expertise as given in Section 2.2. These results extend to the infinite-horizon model in Section 3.

6.1 Public state realizations

Proposition 4 shows that if the state remains hidden after the DM picks his action, then equilibrium predictions differ fundamentally from those in Proposition 2.

Proposition 4. *Consider an extension of the example in Section 2 in which the state remains hidden after the DM takes his action. There is a unique equilibrium where the informed type reports the true state. In this equilibrium, there exists $\hat{\mu} \equiv \hat{\mu}(p) \in (\frac{1}{2}, 1)$ such that:*

1. *A crisis of expertise happens if and only if $\mu \geq \hat{\mu}$.*
2. *$\hat{\mu}(p)$ is strictly increasing in p.*

The intuition is straightforward. If the uninformed type knows that the DM cannot verify whether or not her report is correct or not via a state realization, then her gambling incentive escalates. In turn, the DM dismisses report 1 unless the quality

[14] See "Fake 'Expert' Diminishes the Value of Genuine Financial Help", *The Seattle Times*, August 24, 2019.

of public information is low enough. Contrary to my main insight, namely that high-quality public information sustains the DM's trust in expert advice, the crisis of expertise here is plainly a phenomenon where the DM substitutes public information for expert advice.

In contrast, Proposition 5 below shows that my main insight holds so long as the state is publicly realized with a high enough probability to discipline the uninformed type's gambling:

Proposition 5. *Consider an extension of the example in Section 2 in which the state is publicly realized with some probability $v \in (0,1)$ and is hidden otherwise after the DM takes his action. There is a unique equilibrium where the informed type reports the true state. In this equilibrium, there is $\underline{v} \in (0,1)$ such that for every $v \geq \underline{v}$, there is $\underline{\mu}_v \in (\frac{1}{2}, 1)$ such that if $\mu \geq \underline{\mu}_v$, then the crisis of expertise does not happen.*

Taking the interpretation from the Introduction that a labor market publicly verifies whether or not the expert's report matches the true state, Propositions 4 and 5 imply that for high-quality public information to sustain the DM's trust in expert advice, the labor market must perform sufficiently high-quality audits of the experts.

While Propositions 4 and 5 concern extreme values of the realization probability v, the reader might be be interested in characterizing the crisis for all possible v. This is difficult to achieve analytically but numerical simulation is straightforward. Appendix A.2 provides details.

6.2 Extensions

Finally, I describe how my analysis can be enriched.

Imperfectly informed type. My model has assumed that the informed type's signal perfectly reveals the state. Proposition 6 below shows that my main insight holds so long as the informed type's signal is sufficiently informative about the state. Specifically, extend the example in Section 2 as follows. Fix a constant $\lambda \in (\mu, 1)$. The informed type's signal matches the state with probability λ and does not match the state otherwise. Thus, her signal is imperfect and, to align with the motivation, her signal is more accurate than public information is about the state.

In this extension, truthful reporting by the informed type need not constitute an equilibrium; to align with my motivation and the above analysis, I follow the literature on pooling reputations (Fudenberg and Levine, 1989, 1992) to assume that the informed type commits to report her signal truthfully.

Proposition 6. *Consider an extension of the example in Section 2 as described above. There is a unique equilibrium. In this equilibrium, there exists* $\underline{\lambda} \in (0, 1)$ *such that for every* $\lambda \geq \underline{\lambda}$*, there is* $\bar{\mu}_\lambda \in (\frac{1}{2}, 1)$ *such that if* $\mu \geq \underline{\mu}_\lambda$*, then the DM matches his action with the expert's report.*

If the quality of public information μ is high enough, so is the informed type's signal accuracy λ; the equilibrium outcome then approximates that in Section 2 and this proposition follows.

The reader might be interested in characterizing the crisis for all possible λ. This is difficult to achieve analytically but numerical simulation is straightforward, suggesting a characterization similar to the one in Proposition 2. Appendix A.3 provides details.

Serially correlated states. My results do not hinge on the independence of states across periods, so long as in each period, the DM and the uninformed type share a common prior state belief about the current state, and the expert's reputation is commonly known.

The distribution F from which the quality μ is drawn in each period can thus be extended to depend on past state realizations, for instance. In this case, the uninformed type's equilibrium continuation payoff in each period depends on past state realizations in addition to her reputation, as these realizations determine future state distributions, which in turn determine future DMs' perceived value of receiving advice and the expert's future wages. It is, nonetheless, straightforward to extend the present proofs and show that my results extend.

Uncommon prior beliefs. Finally, I consider an extension where the uninformed type and the DM have different prior state beliefs because, for instance, the DM is inattentive to all available public information about the state.

Specifically, consider an extension of the example in Section 2 in which the DM's prior state belief that $s = 0$ is given by $\mu' \neq \mu$ while the uninformed type's prior state belief that $s = 0$ continues to be given by μ. In this setting, the uninformed type's equilibrium strategy continues to be given by (3), as it is determined by her incentive constraint that is independent of the DM's belief μ'.[15] The DM's problem, in contrast, depends on his own belief μ' and is different from that in Section 2: the DM's optimal expected benefit, if he had to take an action without expert advice, is now given by μ' but not μ, and the action that achieves this benefit need not be action 0.

Proposition 7 below shows that equilibrium predictions in this extension are similar to those in my main analysis if the beliefs μ and μ' agree on which state is more likely to be true, but are fundamentally different otherwise.

Proposition 7. *Consider an extension of the example in Section 2 as described above in which the uninformed type and the DM have different prior state beliefs. There is a unique equilibrium where the informed type reports the true state. In this equilibrium, if $\mu' \geq \frac{1}{2}$, then there is $\underline{\mu} \in (\frac{1}{2}, 1)$ such that the DM matches his action with the expert's report if and only if $\mu > \underline{\mu}$. In contrast, suppose that $\mu' < \frac{1}{2}$. Then the DM matches his action with report 1; he matches his action with report 0 if and only if μ' is sufficiently close to $\frac{1}{2}$.*

The intuition is as follows. If $\mu' \geq \frac{1}{2}$, then action 0 is the DM's optimal action if the expert is uninformed. The DM matches his action with report 1 if and only if he believes that the uninformed type is unlikely to gamble and send report 1, i.e., if and only if μ is high enough. In contrast, if $\mu' < \frac{1}{2}$, then action 1 is the DM's optimal action if the expert is uninformed. Because the uninformed type's strategy (3) is unaffected by μ' and it reports state 0 with a relatively high probability, the DM matches his action with report 0 if and only if his own belief μ' is very noisy.

[15]This observation contrasts with Lai (2014) whose analysis is related to this extension. He studies a version of the partisan cheap talk game à la Crawford and Sobel (1982) where the sender wishes to bias the action of a privately and partially informed receiver. He shows that the receiver's private knowledge can reduce his equilibrium payoff by crowding out the information revealed by the sender.

Appendices

A Omitted details

A.1 Off-path beliefs in Section 2

In Section 2, I have assumed that the DM infers that the expert is uninformed for sure if he receives a report that he expects from neither type. This is the case if the expert sends some report that is assigned probability zero by the uninformed type's strategy and the report turns out to be incorrect. Lemma 4 below shows that a violation of this assumption precludes existence of an equilibrium in which the informed type reports the true state for certain values of (p, μ); moreover, if such an equilibrium exists, then it is determined as in the example.

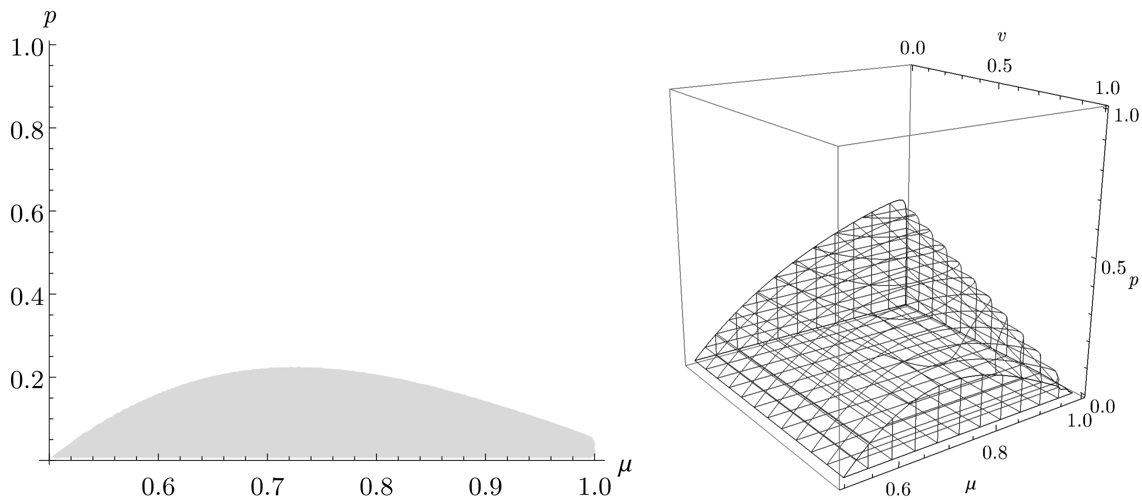
Lemma 4. *There exists (p, μ) such that no equilibrium exists in which the informed type reports the true state and the DM assigns a positive reputation to the expert after the state is realized if he received a report that he expects from neither type according to his conjecture of both types' strategies. For other values of (p, μ), if such an equilibrium exists, then the equilibrium strategies are determined as in Proposition 1.*

The reason why an equilibrium in which the informed type reports the true state might not exist is that if the DM conjectures that the uninformed type never reports a state s, then a positive off-path reputation associated with an incorrect report s creates an extra motive for the uninformed type to deviate from the DM's conjecture to report s; but if the DM conjectures that the uninformed type reports s with some positive probability, then this extra motive to report s disappears. The uninformed type then deviates from the DM's conjecture to not report s.

A.2 Noisy state verification

In this section, I provide the omitted details of the extension in Proposition 5.

For each $v \in (0, 1)$. the unique equilibruim strategy $\tilde{\alpha}^*_{p,\mu}$ can be obtained easily from the conditions (33) and (34) as derived in the proof of Proposition 4. Its expression, nonetheless, is difficult to work with analytically and thus a characterization of the crisis as in Proposition 2 is difficult to achieve. Numerical simulation, nonetheless, is



(a) $v = 0.9$ **(b)** $(p, v) \in (0, 1)^2$

Figure 4: The crisis of expertise with noisy state verification

straightforward. Figure 4 illustrates. Figure 4a plots the region of public beliefs (p, μ) given which the DM chooses action 0 independently of his received report, assuming that $v = 0.9$. For reputations $p \geq 0.05$, $v = 0.9$ is sufficiently high to preempt the crisis when μ is high enough. In contrast, for reputations $p < 0.05$, $v = 0.9$ is not high enough to do so. Figure 4b shows this characterization more generally. It plots the region of public beliefs (p, μ) given which the DM chooses action 0 independently of his received report for each $v \in (0, 1)$. It shows that for higher reputations, the cutoff for v above which high-quality public information sustains trust is lower.

A.3 Imperfectly informed type

In this section, I provide the omitted details of the extension in Proposition 6.

In this extension, the unique equilibruim strategy $\tilde{\alpha}^*_{p,\mu}$ that solves (35), as derived in the proof of Proposition 6, can be easily obtained. But, just as in Appendix A.2, the resulting expression is difficult to work with analytically and so a characterization of the crisis as in Proposition 1 is difficult to achieve. Numerical simulation, nonetheless, is straightforward and suggests a characterization of the crisis similar to that in Proposition 2. Figure 5 illustrates by plotting the values of $\mu \in [\frac{1}{2}, 1)$ and $\lambda \in (\mu, 1)$ given which the DM chooses action 0 independently of his received report. This figure demonstrates that for any $p \in (0, 1)$, there is a cutoff for the signal precision λ above which public information with sufficiently high quality preempts the crisis. It also

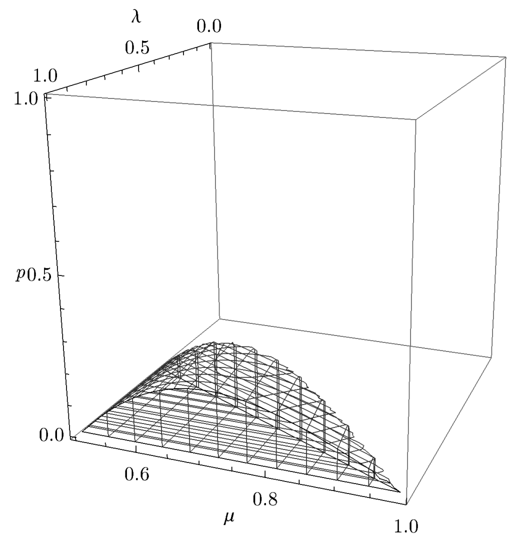


Figure 5: The crisis of expertise with imperfectly informed type

shows that this cutoff is lower given a higher value of p.

B Proofs

B.1 Proof of Proposition 1

Consider the uninformed type's strategy. Her equilibrium strategy is to report state 0 if $p\mu \geq 1-\mu$. Indeed, if the DM conjectures that the uninformed type plays this strategy, then from the uninformed type's perspective, her expected payoff by reporting 0 is $p\mu$. If she deviates to report state 1, then her expected payoff is $1-\mu$. This deviation is not profitable if $p\mu \geq 1-\mu$. Conversely, if $1-\mu > p\mu$, then the DM must conjecture that the uninformed type reports both states with some positive probabilities $\hat{\alpha}$ and $1-\hat{\alpha}$ such that the uninformed type is indifferent between reporting the two states:

$$\mu\left(\frac{p}{p+(1-p)\hat{\alpha}}\right) = (1-\mu)\left(\frac{p}{p+(1-p)(1-\hat{\alpha})}\right).$$

In equilibrium, the DM's conjecture must be correct, and so the uninformed expert must choose report probability $\alpha^*_{p,\mu}$, as given in (3).

Given the DM's conjecture of the uninformed type's equilibrium report probability $\alpha^*_{p,\mu}$, the DM optimally matches his action with report 0 if and only if his payoff from doing so exceeds that from mismatching the report:

$$\frac{p\mu}{p\mu + (1-p)\alpha^*_{p,\mu}} + \frac{(1-p)\alpha^*_{p,\mu}}{p\mu + (1-p)\alpha^*_{p,\mu}} \cdot \mu \geq \frac{(1-p)\alpha^*_{p,\mu}}{p\mu + (1-p)\alpha^*_{p,\mu}}(1-\mu).$$

This inequality holds because $p \in (0,1]$, $\mu \in [\frac{1}{2}, 1)$ and $\alpha^*_{p,\mu} \in [0,1]$. On the other hand, he optimally matches his action with report 1 if and only if his payoff from matching his action with the expert's report exceeds that from mismatching it:

$$\frac{p(1-\mu)}{p(1-\mu) + (1-p)(1-\alpha^*_{p,\mu})} + \frac{(1-p)(1-\alpha^*_{p,\mu})}{p(1-\mu) + (1-p)(1-\alpha^*_{p,\mu})}(1-\mu)$$
$$> \frac{(1-p)(1-\alpha^*_{p,\mu})}{p(1-\mu) + (1-p)(1-\alpha^*_{p,\mu})} \cdot \mu.$$

This inequality simplifies to $\alpha^*_{p,\mu} > \kappa_{p,\mu}$, where $\kappa_{p,\mu}$ is given in (2), as desired.

B.2 Proof of Proposition 2

By Proposition 1, the DM optimally takes action 0 independently of the expert's report if and only if $\alpha^*_{p,\mu} - \kappa_{p,\mu} \leq 0$, where $\kappa_{p,\mu}$ is given in (2). The left side of this inequality is strictly convex in μ. It has no root if $p > (\sqrt{2}-1)/2$, and has roots

$$\frac{3 + 2p - \sqrt{1 - 4p(p+1)}}{4(p+1)}, \quad \frac{3 + 2p + \sqrt{1 - 4p(p+1)}}{4(p+1)} \in \left(\frac{1}{2}, 1\right)$$

otherwise. Setting

$$p^* := \frac{1}{2}(\sqrt{2} - 1), \tag{11}$$

$$\underline{\mu}^*(p) := \begin{cases} \dfrac{3 + 2p - \sqrt{1 - 4p(p+1)}}{4(p+1)}, & \text{if } p < p^*, \\ \dfrac{1}{\sqrt{2}}, & \text{otherwise}, \end{cases} \tag{12}$$

$$\bar{\mu}^*(p) := \begin{cases} \dfrac{3 + 2p + \sqrt{1 - 4p(p+1)}}{4(p+1)}, & \text{if } p < p^*, \\ \dfrac{1}{\sqrt{2}}, & \text{otherwise}, \end{cases} \tag{13}$$

completes the proof; it is straightforward to verify that $\underline{\mu}^*(p)$ is strictly increasing on $[0, p^*)$, and $\bar{\mu}^*(p)$ is strictly decreasing on $[0, p^*)$.

B.3 Proof of Lemma 1

Suppose that a reputational equilibrium exists, and fix this equilibrium. Let $\mathcal{V}^U(h^U)$ denote the uninformed type's (continuation) payoff at history h^U where all past reports are correct in this equilibrium. I prove each part of the proposition in order.

1(a). It is clear from Bayes' rule that in any reputational equilibrium, both types' continuation payoffs upon sending an incorrect report are zero if this report is sent when the expert's reputation is short of unity, as discussed in the main text. I defer the remainder of the proof of this part to 2(a) below.

1(b). In any period in which $p = 0$, irrespective of the expert's report, the DM's expected payoff from choosing action 0 is μ and that from choosing action 1 is $1-\mu$. Because $\mu \geq \frac{1}{2}$ and the (innocuous) tie-breaking rule that the DM chooses action 0 whenever he is indifferent between actions 0 and 1, the claim follows.

2(a). (i). Reputational payoff and monotonicity:

Given any two histories $h^U, \tilde{h}^U$ with identical associated reputation $p < 1$, in any reputational equilibrium, it must hold that $\mathcal{V}^U(h^U) = \mathcal{V}^U(\tilde{h}^U) =: V^U(p)$ for some function $V^U : [p_0, 1) \to \mathbf{R}_+$, for otherwise the uninformed type has a profitable deviation at either h^U or $\tilde{h}^U$. The lower bound p_0 of the domain of V^U follows from the fact that in equilibrium, in each period with no incorrect past report, the expert's reputation is at least the prior reputation p_0. Finally, by Property 2, V^U is strictly increasing.

In the end of this proof, I return to this part and show that the domain of V^U can be extended to $[p_0, 1]$.

(ii). Payoff continuity:

I next show that $V^U(p)$ is continuous in $p \in [p_0, 1)$. Suppose, towards a contradiction, $V^U(p)$ is discontinuous at some $p^\dagger \in [p_0, 1)$. Because V^U is strictly increasing, for each $p \in [p_0, p^\dagger]$, there exists $\hat{\alpha}^\dagger \in (0,1]$ such that

$$\frac{p}{p+(1-p)\hat{\alpha}^\dagger} = p^\dagger, \tag{14}$$

$$\lim_{\hat{\alpha}\uparrow\hat{\alpha}^\dagger} V^U\left(\frac{p}{p+(1-p)\hat{\alpha}}\right) > \lim_{\hat{\alpha}\downarrow\hat{\alpha}^\dagger} V^U\left(\frac{p}{p+(1-p)\hat{\alpha}}\right). \tag{15}$$

Moreover, by Property 2, at any uninformed type's history $\hat{h}^U$ with associated reputation $p \in [p_0, 1)$, there is a set of state beliefs μ with positive measure such that her incentive constraint for reporting 0 with probability one in this period is violated:

$$\mu V^U(p) < (1-\mu)v^{\theta_U}(1; h^U_{r=1}), \tag{16}$$

where v^{θ_U} is defined in Section 3, $h^U_{r=1}$ is the concatenation of history $\hat{h}^U$ followed by a correct report 1. In particular, given $\hat{\alpha}^\dagger$, because V^U is strictly increasing, there is one such μ given which (16) holds and

$$\lim_{\hat{\alpha}\uparrow\hat{\alpha}^\dagger} \mu V^U\left(\frac{p}{p+(1-p)\hat{\alpha}}\right) - (1-\mu)V^U\left(\frac{p}{p+(1-p)(1-\hat{\alpha})}\right) > 0, \tag{17}$$

$$\lim_{\hat{\alpha}\downarrow\hat{\alpha}^\dagger} \mu V^U\left(\frac{p}{p+(1-p)\hat{\alpha}}\right) - (1-\mu)V^U\left(\frac{p}{p+(1-p)(1-\hat{\alpha})}\right) < 0. \tag{18}$$

In a period following this history with this pair of beliefs (p, μ), the uninformed type must report 0 with some probability $\hat{\alpha}^\dagger < 1$ such that she is indifferent between reporting 0 and reporting 1, but (17) and (18) together ensure that no such $\hat{\alpha}^\dagger$ exists, yielding a contradiction as desired.

(iii). Strategy:

I next show that in each period following some history of play that leads to public beliefs (p, μ) and there is no incorrect past report, the uninformed type's strategy is characterized by (p, μ).

I first show that this is true in the case of $p < 1$. In this period, suppose that the uninformed type's history is given by some h^U. Then the uninformed type's report probability, denoted by α_{h^U}, must satisfy

$$\alpha_{h^U} \in \arg\max_{\hat{\alpha}_{h^U}\in[0,1]} \left[\mu\hat{\alpha}_{h^U} V^U\left(\frac{p}{p+(1-p)\alpha_{h^U}}\right) \right. \\ \left. + (1-\mu)(1-\hat{\alpha}_{h^U})V^U\left(\frac{p}{p+(1-p)(1-\alpha_{h^U})}\right)\right], \tag{19}$$

where the reputations upon correctly reporting 0 and correctly reporting 1 are computed based on the DMs' equilibrium conjecture that the uninformed type's strategy in this period is α_{h^U}. Since there is no incorrect past report in this period, $p \geq p_0 > 0$. Moreover, since V^U is strictly increasing, α_{h^U} must be such that, for some cutoff $\mu_\dagger(p) \in (\frac{1}{2}, 1)$,

$$\alpha_{h^U} = \begin{cases} 1, & \text{if } \mu \geq \mu_\dagger(p), \\ \alpha^\dagger_{p,\mu}, & \text{otherwise,} \end{cases}$$

where $\alpha^\dagger_{p,\mu}$ uniquely solves

$$\mu V^U\left(\frac{p}{p+(1-p)\alpha^\dagger_{p,\mu}}\right) = (1-\mu)V^U\left(\frac{p}{p+(1-p)(1-\alpha^\dagger_{p,\mu})}\right). \tag{20}$$

Thus, this strategy α_{h^U} depends only on the beliefs (p, μ), and I write α_{h^U} as $\alpha_{p,\mu}$. In turn, by (5), the equilibrium wage in this period is plainly a function of (p, μ). In the remainder of this proof, I denote this wage by $w_{p,\mu}$; it is given by

$$w_{p,\mu} = \max\left[p + (1-p)(\alpha_{p,\mu}\mu + (1-\alpha_{p,\mu})(1-\mu)) - \mu, 0\right]. \tag{21}$$

Because V^U is strictly increasing, (20) implies that there exists $\mu_\dagger \equiv \mu_\dagger(p)$ such that $\alpha_{p,\mu}$ is strictly increasing in μ on $(\frac{1}{2}, \mu_\dagger]$ and is 1 if $\mu \in (\mu_\dagger, 1]$. Moreover, by (20), $\alpha_{p,\frac{1}{2}} = \frac{1}{2}$. Thus, $\alpha_{p,\mu} \in [\frac{1}{2}, 1]$.

1(a) (Continued.) I now return to Part 1(a) and show that in any reputational equilibrium, both types' payoffs upon reporting incorrectly when the reputation is one are zero. In the following, I show that this is the case for the uninformed type, which implies that this must also be the case for the informed type. This is because, if the uninformed type's payoff is zero in this continuation, then the DM's wage payment in each period in this continuation must be zero, irrespective of the continuation history.

Suppose, towards a contradiction, that there exists a reputational equilibrium in which in a period following some uninformed type's off-path history h^U, in which no past report is incorrect, the expert's reputation is $p = 1$ and the state belief is μ. By Property 2,

$$\lim_{p \uparrow 1} V^U(p) \leq v^{\theta_U}(1; h^U).$$

If the uninformed type's payoff remains positive after sending an incorrect report of either 0 or 1 in this period, then there must exist a discontinuity:

$$\lim_{p \uparrow 1} V^U(p) < v^{\theta_U}(1; h^U). \tag{22}$$

Now, fix an uninformed type's history $\hat{h}^U$ given which no past report is incorrect, the reputation is $p < 1$. Because V^U is strictly increasing, there is a positive measure of state beliefs μ given which

$$\mu V^U(p) - (1-\mu) v^{\theta_U}(1; h^U_{r=1}) < 0, \tag{23}$$

where $h^U_{r=1}$ denote a concatenation of history $\hat{h}^U$ followed by a correct report 1, and

$$\lim_{\hat{\alpha} \uparrow 1} \mu V^U\left(\frac{p}{p + (1-p)\hat{\alpha}}\right) - (1-\mu) V^U\left(\frac{p}{p + (1-p)(1-\hat{\alpha})}\right) > 0. \tag{24}$$

By (23), in this period given these beliefs (p, μ), the uninformed type must report 0 with some probability $\alpha^\dagger < 1$ and must be indifferent between reporting the two states. But by (24), and because V^U is strictly increasing, no such $\alpha^\dagger$ exists, yielding a contradiction as desired.

To sum up, in a period with reputation $p = 1$, irrespective of the history of play that leads to this reputation, if the uninformed type reports incorrectly, then her payoff is zero. Thus, given any two histories $h^U, \tilde{h}^U$ with identical associated reputation $p = 1$, in any reputational equilibrium, $\mathcal{V}^U(h^U) = \mathcal{V}^U(\tilde{h}^U) =: \hat{V}^U(1)$, for some function $\hat{V}^U$. Moreover, $\hat{V}^U(1) > V^U(p)$ for each $p \in [p_0, 1)$ by Property

2, and

$$\lim_{p\uparrow 1} V^U(p) = \hat{V}^U(1)$$

by continuity. Thus, the domain of the function V^U can be extended from $[0,1)$ to $[0,1]$ such that $V^U(1) := \hat{V}^U(1)$, and $V^U : [p_0, 1] \to \mathbf{R}_+$ is continuous and strictly increasing.

2(a). (Continued.) I next return to Part 2(a) to show that the uninformed expert's strategy at a history h^U given which the public beliefs are (p, μ), with $p = 1$, and there is no past report, is also characterized by the beliefs (p, μ). This must be the case since an incorrect report in this period leads to zero payoff for the expert in view of the above derivation, and therefore the uninformed type's report probability in this period is also characterized by the solution to (19).

2(b). That the informed type reports the true state follows from Property 1. Next, her wage $w_{p,\mu}$ in each period without an incorrect report is characterized by the public beliefs (p, μ) and given in (21), irrespective of her private history h^I. Thus, her wage in any such period prior to μ being drawn is completely determined by p. Because the informed type never reports incorrectly on path, it follows that her payoff at any history on path with associated reputation $p \in [p_0, 1]$ is a function $V^I(p)$. This function is strictly increasing by Property 2. Finally, this function is continuous. The reason is that $\alpha_{p,\mu}$ as determined by the continuous function V^U, as seen in Part 2(a), is continuous in p, so that the informed type's wage $w_{p,\mu}$ in each period on path is continuous in p.

2(c). This is proved in Proposition 1.

B.4 Proof of Lemma 2

I prove the claims in this lemma in order.

1. Both parts (a) and (b) are proved in Part 2(a) of Lemma 1.

2. To prove this part, it is instructive to first prove the following claim. Let $\{\delta_n\}_{n=0}^{\infty}$ be an increasing sequence of discount factors converging to one. Let V_n^U denote the uninformed type's continuation payoff function in the reputational equilibrium when the discount factor is δ_n.

Claim 1. *For every $\varepsilon > 0$, there exists $N > 0$ such that for every $n \geq N$,*

$$\sup_{p\in[p_0,1]} \left|\varepsilon p - V_n^U(p)\right| < \varepsilon, \tag{25}$$

$$\sup_{p\in[p_0,1]} \left|V_n^U(p)\right| < \varepsilon. \tag{26}$$

Proof of Claim 1. Fix $\varepsilon > 0$. In the equilibrium, because the uninformed type sends an incorrect report with positive probability, in which case she receives zero continuation payoff, there exists N such that for every $n \geq N$, for every $p \in [p_0, 1]$, $0 < V_n^U(p) < \varepsilon p$. Thus,

$$0 < V_n^U(p) < \varepsilon,$$

giving (26), and because $V_n^U(p) \geq 0$,

$$0 < \varepsilon p - V_n^U(p) < \varepsilon p \leq \varepsilon,$$

giving (25). ■

Let α^* and α^n be as defined in Section 4. Fix $\varepsilon > 0$. Fix some $N := \max(N', N'', N''')$, with N', N'', N''' chosen below, and fix some $n \geq N$. I prove 2(a)—(c) in order.

(a) Define $f : [0,1] \to \mathbf{R}$ such that

$$f(e) := \mu \cdot \frac{p}{p + (1-p)e} - (1-\mu) \cdot \frac{p}{p + (1-p)(1-e)}.$$

This function is strictly decreasing and continuously differentiable. To show (7), fix $(p, \mu) \in [p_0, 1] \times [\frac{1}{2}, 1)$. By (25), for each $p \in [p_0, 1]$, $V_n^U(p)$ can be written as $V_n^U(p) = \eta_n\,(p + z_n(p))$ for some constant $\eta_n > 0$ and some function $z_n : [p_0, 1] \to \mathbf{R}$ such that $(\eta_n)_{n=0}^\infty$ converges to zero and $(z_n)_{n=0}^\infty$ converges uniformly to zero.

Suppose first that $\alpha^n_{p,\mu} = 1$. Then, by Lemma 1, the following uninformed type's incentive constraint must hold in the reputational equilibrium:

$$\mu V_n^U(p) \geq (1-\mu)V_n^U(1) \quad \Longrightarrow \quad f(1) \geq (1-\mu)z_n(1) - \mu z_n(p). \tag{27}$$

If $(1-\mu)z_n(1) - \mu z_n(p) \geq 0$, then $f(1) \geq 0$ and so by Proposition 1, $\alpha^*_{p,\mu} = 1$. This implies that $|\alpha^*_{p,\mu} - \alpha^n_{p,\mu}| = 0$. Suppose instead that $(1-\mu)z_n(1) - \mu z_n(p) < 0$. There are two cases:

i. $f(1) \geq 0$. In this case, by Proposition 1, $\alpha^*_{p,\mu} = 1$. This implies that $|\alpha^*_{p,\mu} - \alpha^n_{p,\mu}| = 0$.

ii. $f(1) < 0$. In this case, by Proposition 1, $\alpha^*_{p,\mu} < 1$ and it is characterized by the uninformed type's incentive constraint

$$f(\alpha^*_{p,\mu}) = 0. \tag{28}$$

Because f is continuous and strictly decreasing, there exists $\zeta > 0$ such that $|f(1) - f(\alpha^*_{p,\mu})| < \zeta$ if and only if $|1 - \alpha^*_{p,\mu}| < \varepsilon$. Fix this ζ. Because $(z_n)_{n=0}^{\infty}$ converges uniformly to zero, there exists $N' > 0$ such that for every $n \geq N'$,

$$|(1-\mu)z_n(1) - \mu z_n(p)| < \frac{\zeta}{2}.$$

This implies that

$$\frac{\zeta}{2} > f(\alpha^*_{p,\mu}) = 0 > f(1) \geq -\frac{\zeta}{2}, \tag{29}$$

where the equality follows from (28) and the last inequality follows from (27). Thus, $\left|f(\alpha^*_{p,\mu}) - f(1)\right| < \zeta$, and therefore $|\alpha^*_{p,\mu} - 1| < \varepsilon$, as claimed. Because $\alpha^n_{p,\mu} = 1$, $|\alpha^*_{p,\mu} - \alpha^n_{p,\mu}| < \varepsilon$, as desired.

Finally, suppose that $\alpha^n_{p,\mu} < 1$. Then $\alpha^n_{p,\mu}$ is characterized by the uninformed type's incentive constraint

$$\mu V^U_n\left(\frac{p}{p+(1-p)\alpha^n_{p,\mu}}\right) - (1-\mu)V^U_n\left(\frac{p}{p+(1-p)(1-\alpha^n_{p,\mu})}\right) = 0$$
$$\iff \qquad f(\alpha^n_{p,\mu}) = (1-\mu)z_n(1) - \mu z_n(p).$$

If $\alpha^*_{p,\mu} = 1$, the proof of case (ii) above applies, so that $|\alpha^*_{p,\mu} - \alpha^n_{p,\mu}| < \varepsilon$, as desired. Suppose that $\alpha^*_{p,\mu} < 1$ instead. Fix this $\alpha^*_{p,\mu}$. Because f is continuous and strictly decreasing, there exists $\zeta' > 0$ such that $|f(\alpha^n_{p,\mu}) - f(\alpha^*_{p,\mu})| < \zeta'$ if and only if $|\alpha^n_{p,\mu} - \alpha^*_{p,\mu}| < \varepsilon$. Now, $\alpha^*_{p,\mu}$ must satisfy (28), so that there exists $N'' > 0$ such that for every $n \geq N''$,

$$|f(\alpha^n_{p,\mu}) - f(\alpha^*_{p,\mu})| = |(1-\mu)z_n(1) - \mu z_n(p)| < \zeta'.$$

Thus, $|\alpha^*_{p,\mu} - \alpha^n_{p,\mu}| < \varepsilon$, as desired.

(b) I next show (8). For each pair $(p,\mu) \in [p_0,1] \times [\frac{1}{2},1)$, write

$$\alpha^n_{p,\mu} = \alpha^*_{p,\mu} + \gamma^n_{p,\mu}$$

for some function $\gamma^n : [p_0,1] \times [\frac{1}{2},1) \to \mathbf{R}$. By Part (a) above, $(\gamma^n)_{n=0}^{\infty}$ converges uniformly to zero. Because the sup-norm is continuous,

$$\begin{aligned} &\lim_{n\to\infty} \sup_{(p,\mu,\mu')\in[p_0,1]\times[\frac{1}{2},1)^2:\mu\neq\mu'} \left| \frac{\gamma^n_{p,\mu'} - \gamma^n_{p,\mu}}{\mu'-\mu} \right| \\ &= \sup_{(p,\mu,\mu')\in[p_0,1]\times[\frac{1}{2},1)^2:\mu\neq\mu'} \left| \lim_{n\to\infty} \frac{\gamma^n_{p,\mu'} - \gamma^n_{p,\mu}}{\mu'-\mu} \right| = 0. \end{aligned}$$

Thus, there exists $N'' > 0$ such that for every $n \geq N''$,

$$\sup_{(p,\mu,\mu')\in[p_0,1]\times[\frac{1}{2},1)^2:\mu\neq\mu'} \left| \frac{\gamma^n_{p,\mu'} - \gamma^n_{p,\mu}}{\mu'-\mu} \right| < \varepsilon.$$

Fix this N'' and then fix some $n \geq N''$. It follows that

$$\begin{aligned} &\sup_{(p,\mu,\mu')\in[p_0,1]\times[\frac{1}{2},1)^2:\mu\neq\mu'} \left| \frac{\alpha^n_{p,\mu'} - \alpha^n_{p,\mu}}{\mu'-\mu} - \frac{\alpha^*_{p,\mu'} - \alpha^*_{p,\mu}}{\mu'-\mu} \right| \\ &= \sup_{(p,\mu,\mu')\in[p_0,1]\times[\frac{1}{2},1)^2:\mu\neq\mu'} \left| \frac{\gamma^n_{p,\mu'} - \gamma^n_{p,\mu}}{\mu'-\mu} \right| < \varepsilon, \end{aligned}$$

as desired.

(c) I next show (8). Fix $N''' := \max(N'''_{(1)}, N'''_{(2)}, N'''_{(3)})$, where $N'''_{(1)}, N'''_{(2)}$, and $N'''_{(3)}$ are chosen below., fix $p, p' \in [p_0, 1]$ with $p' > p$. The objective is to show that for each $n \geq N'''$, $\alpha^n_{p',\mu} \geq \alpha^n_{p,\mu}$. I consider two cases. Suppose first that $\alpha^n_{p,\mu} = 1$. Then the uninformed type's incentive constraint requires that $\mu V^U_n(p) \geq (1-\mu)V^U(1)$. Because V^U_n is strictly increasing, $\mu V^U(p') > (1-\mu)V^U(1)$. This gives $\alpha^n_{p',\mu} = 1$, and so $\alpha^n_{p',\mu} \geq \alpha^n_{p,\mu}$.

Suppose instead $\alpha^n_{p',\mu} < 1$. If $\alpha^n_{p',\mu} = 1$, then it follows immediately that $\alpha^n_{p',\mu} > \alpha^n_{p,\mu}$. Thus, consider the case $\alpha^n_{p',\mu} < 1$. As in Part (b), write

$$\alpha^n_{p,\mu} = \alpha^*_{p,\mu} + \gamma^n_{p,\mu},$$
$$\alpha^n_{p',\mu} = \alpha^*_{p',\mu} + \gamma^n_{p',\mu},$$

for some function $\gamma^n : [p_0, 1] \times [\frac{1}{2}, 1) \to \mathbf{R}$. By Part (a), $(\gamma^n)^\infty_{n=0}$ converges uniformly to zero. There are two sub-cases:

i. $\alpha^*_{p',\mu} < 1$. In this case, the uninformed type's incentive constraint for reporting state 0 with probability one must be violated at reputation p', i.e., $\mu V^U(p') < (1-\mu)V^U(1)$. Because V^U is strictly increasing, it follows that $\mu V^U(p) < (1-\mu)V^U(1)$. Thus, $\alpha^*_{p,\mu} < 1$. Then

$$\left| \frac{\alpha^n_{p',\mu} - \alpha^n_{p,\mu}}{p'-p} - \frac{\alpha^*_{p',\mu} - \alpha^*_{p,\mu}}{p'-p} \right| = \left| \frac{\gamma^n_{p',\mu} - \gamma^n_{p,\mu}}{p'-p} \right| < \varepsilon.$$

Because

$$\frac{\alpha^*_{p',\mu} - \alpha^*_{p,\mu}}{p'-p} = \frac{2\mu - 1}{(1-p')(1-p)} > 0,$$

it follows that there exists $N'''_{(1)} > 0$ such that for every $n \geq N'''_{(1)}$, $\alpha^n_{p',\mu} > \alpha^n_{p,\mu}$.

ii. $\alpha^*_{p',\mu} = 1$ and $\alpha^*_{p,\mu} = 1$. Given n, because $\alpha^n_{p,\mu} < 1$ and $\alpha^n_{p',\mu} < 1$, there exists $\tilde{N} > 0$ such that for every $\tilde{n} \geq \tilde{N}$, $\alpha^{\tilde{n}}_{p',\mu} - \alpha^n_{p,\mu} > 0$ and

$$\frac{\alpha^{\tilde{n}}_{p',\mu} - \alpha^n_{p,\mu}}{p'-p} > 0.$$

Thus, there exists $N'''_{(2)} > \tilde{N}$ such that for every $n \geq N'''_{(2)}$,

$$\left| \frac{\alpha^n_{p',\mu} - \alpha^n_{p,\mu}}{p'-p} - \frac{\alpha^{\tilde{n}}_{p',\mu} - \alpha^n_{p,\mu}}{p'-p} \right| = \left| \frac{\gamma^n_{p',\mu} - \gamma^n_{p,\mu}}{p'-p} - \frac{\gamma^{\tilde{n}}_{p',\mu} - \gamma^n_{p,\mu}}{p'-p} \right| < \varepsilon,$$

and so $\alpha^n_{p',\mu} > \alpha^n_{p,\mu}$.

iii. $\alpha^*_{p',\mu} = 1$ and $\alpha^*_{p,\mu} < 1$. Then

$$\frac{\alpha^*_{p',\mu} - \alpha^*_{p,\mu}}{p' - p} > 1,$$

and there exists $N'''_{(3)} > 0$ such that for every $n \geq N'''_{(3)}$,

$$\left| \frac{\alpha^n_{p',\mu} - \alpha^n_{p,\mu}}{p' - p} - \frac{\alpha^*_{p',\mu} - \alpha^*_{p,\mu}}{p' - p} \right| = \left| \frac{\gamma^n_{p',\mu} - \gamma^n_{p,\mu}}{p' - p} \right| < \varepsilon,$$

and so $\alpha^n_{p',\mu} > \alpha^n_{p,\mu}$.

B.5 Proof of Lemma 3

Let $\{\delta_n\}_{n=0}^{\infty}$ and $\alpha^n : [p_0, 1] \times [\frac{1}{2}, 1) \to [0, 1]$ be defined as in Section 4. Let

$$w^n_{p,\mu} = \max[p + (1-p)(\mu\alpha^n_{p,\mu} + (1-\mu)(1-\alpha^n_{p,\mu}) - \mu, 0] \tag{30}$$

denote the expert's wage in a period with no incorrect past report and beliefs (p, μ) in the reputational equilibrium given discount factor δ_n. Then, this wage is equal to 0 if $\mu \in [\underline{\mu}^n, \bar{\mu}^n]$, and is equal to

$$p + (1-p)(\mu\alpha^n_{p,\mu} + (1-\mu)(1-\alpha^n_{p,\mu}) - \mu \tag{31}$$

otherwise. By Part 3 of Lemma 2, there is $N > 0$ such that for every $n \geq N$, $\alpha^n_{p,\mu}$ is increasing in p and so the latter expression is strictly increasing in p.

B.6 Proof of Proposition 3

B.6.1 Existence

In view of Lemma 1, characterizing a reputational equilibrium amounts to describing the players' strategies in each period with no incorrect past report. This is because in any such period, the informed type reports correctly and the uninformed type's strategy determines the DM's best reply.

Thus, any reputational equilibrium is characterized by some report function $\alpha : [p_0, 1] \times [\frac{1}{2}, 1) \to [\frac{1}{2}, 1]$, determining the uninformed type's strategy $\alpha_{p,\mu}$ in each period with beliefs (p, μ) given that there is no incorrect past report. Conversely, for every report function α, fixing the uninformed type's behavior as given by α, there exists an equilibrium (among other players) if Property 2 holds; moreover, in every such equilibrium, at the end of each period when the state is publicly realized, if there is no incorrect past report, then the uninformed type's payoff is a continuous and strictly increasing function $V^U(p)$ of the reputation p, where $V^U : [p_0, 1] \to \mathbf{R}$ is induced by the report function α.

I now show that an equilibrium exists and satisfies Properties 1 and 3. I then verify that if the discount factor is sufficiently close to one, this equilibrium also satisfies Property 2, and so is a reputational equilibrium.

An equilibrium that satisfies Properties 1 and 3 exists. To prove this, it suffices to show that an equilibrium preferred by the uninformed type exists, i.e., there exists a report function that maximizes the uninformed type's *ex ante* payoff, and then to verify that when the DMs conjecture the uninformed type's play follows this report function, the uninformed type has no profitable deviation.

To emphasize that the function $V^U(\cdot)$ is induced by some report function α, I write $V^U(\cdot)$ as $V^U(\cdot; \alpha)$. Let $L^\infty([p_0, 1] \times [\frac{1}{2}, 1); \mathbf{R})$ be the set of Lebesgue measurable functions that maps from $[p_0, 1] \times [\frac{1}{2}, 1)$ to $\mathbf{R}_+$. Let $L_B = \{\alpha \in L^\infty([p_0, 1] \times [\frac{1}{2}, 1); \mathbf{R}) : \|\alpha\|_\infty \leq 1\}$ denote the unit ball. The uninformed type's payoff in an equilibrium (satisfying Properties 1 and 3) that is best for her is the value of the optimization problem below, maximizing her payoff over report functions that the uninformed type plays and the DMs correctly conjecture the uninformed type to play:

$$\sup_{\hat{\alpha} \in L_B} V^U(p_0; \hat{\alpha}). \tag{$*$}$$

The maximum in ($*$) is attained, because the objective is weak-* continuous in α, and L_B is weak-* compact.[16] Let α denote a solution to ($*$).

It remains to show that α indeed constitutes the uninformed type's best reply when the DMs conjecture that she chooses report function α. Specifically, I show that there exists $\alpha_{p,\mu} \in [0, 1]$ that satisfies (6) for each (p, μ) with $p \in [p_0, 1]$. To show this, because $V^U(\cdot; \alpha)$ is strictly increasing, it suffices to show that $V^U(p; \alpha)$ is indeed continuous in $p \in [p_0, 1]$. To this end, I show that for each $p \in [p_0, 1]$, the payoff

[16] As $L^\infty([p_0, 1] \times (\frac{1}{2}, 1]; \mathbf{R})$ is the dual of $L^1([p_0, 1] \times (\frac{1}{2}, 1]; \mathbf{R})$, L_B is weak-* compact by Alaoglu's theorem (e.g., Aliprantis and Border, 2006, Theorem 6.21).

$V^U(p;\alpha)$ is the unique fixed point of the contraction T^U that maps from the set of continuous functions $f : [p_0, 1] \to \mathbf{R}_+$ to itself:

$$T^U f(p) = \max_{\hat{\alpha}_{p,\cdot}:[\frac{1}{2},1)\to[0,1]} \mathbf{E}^F \Bigg[(1-\delta)w_{p,\mu} + \delta\mu\hat{\alpha}_{p,\mu} f\left(\frac{p}{p+(1-p)\alpha_{p,\mu}}\right) \\ + \delta(1-\mu)(1-\hat{\alpha}_{p,\mu}) f\left(\frac{p}{p+(1-p)(1-\alpha_{p,\mu})}\right)\Bigg],$$

where $w_{\cdot,\cdot}$ is the wage induced by α given that there is no incorrect past report, and the expectation $\mathbf{E}^F$ is taken over the set of state beliefs μ in the period according to the distribution F. Observe that the report function α that the DMs expect the uninformed type to play must be continuous in p on (p,μ): by Lemma 1, if α characterizes a reputational equilibrium, then the $V^U(p;\alpha)$ must be continuous in $[p_0, 1]$. Because $\alpha_{p,\mu}$ uniquely solves (6) for each pair (p,μ), it must also be continuous in on $[p_0, 1] \times [\frac{1}{2}, 1)$.

Because $\alpha_{p,\mu}$ is continuous in $(p,\mu) \in [p_0, 1] \times [\frac{1}{2}, 1)$, so is $w_{p,\mu}$. Now, the objective on the right side is weak-* continuous in $\hat{\alpha}_{p,\cdot}$ and is continuous in p, and again the set of functions $\hat{\alpha}_{p,\cdot} : [\frac{1}{2}, 1) \to [0,1]$ is weak-* compact. Moreover, the solution to this objective is single-valued. By the theorem of the maximum (e.g., Stokey, Lucas and Prescott, 1989, Theorem 3.6, p. 62), $V^U(p;\alpha)$ is continuous in p on $[p_0, 1]$. This completes the proof.

With low discounting, a reputational equilibrium exists. I next show that the above equilibrium that exists, which satisfies Properties 1 and 3, also satisfies Property 2 and is therefore a reputational equilibrium, when discounting is low. Let $\{\delta_n\}_{n=0}^{\infty}$, α^*, and α^n be defined as in Section 4.

Claim 2 below is essential.

$$g^n_{p,\mu} := \frac{p}{p+(1-p)\alpha^n_{p,\mu}}, \qquad \text{and} \qquad g^*_{p,\mu} := \frac{p}{p+(1-p)\alpha^*_{p,\mu}}.$$

Note that by (3), $g^*_{p,\mu}$ is strictly increasing in p.

Claim 2. *For every $\varepsilon > 0$, there exists N such that for every $n \geq N$:*

$$\sup_{(\mu,p,p')\in[\frac{1}{2},1)\times[p_0,1]^2:p\neq p'} \left| \frac{g^n_{p,\mu} - g^n_{p',\mu}}{p-p'} - \frac{g^*_{p,\mu} - g^*_{p',\mu}}{p-p'} \right| < \varepsilon. \tag{32}$$

Proof of Claim 2. For each pair $(p, \mu) \in [p_0, 1] \times [\frac{1}{2}, 1)$, write

$$g^n_{p,\mu} = g^*_{p,\mu} + \xi^n_{p,\mu}$$

for some $\xi^n : [p_0, 1] \times [\frac{1}{2}, 1) \to \mathbf{R}$. By Part 1 of Lemma 1, $(\xi^n)_{n=0}^{\infty}$ converges uniformly to zero. Because the sup-norm is continuous,

$$\begin{aligned} &\lim_{n\to\infty} \sup_{(\mu,p,p')\in[\frac{1}{2},1)\times[p_0,1]^2: p\neq p'} \left| \frac{g^n_{p,\mu} - g^n_{p',\mu}}{p - p'} - \frac{g^*_{p,\mu} - g^*_{p',\mu}}{p - p'} \right| \\ = &\sup_{(\mu,p,p')\in[\frac{1}{2},1)\times[p_0,1]^2: p\neq p'} \left| \lim_{n\to\infty} \frac{\xi^n_{p,\mu} - \xi^n_{p',\mu}}{p - p'} \right| = 0. \end{aligned}$$

Thus, there exists $N'' > 0$ such that for every $n \geq N''$,

$$\sup_{(\mu,p,p')\in[\frac{1}{2},1)\times[p_0,1]^2: p\neq p'} \left| \frac{\xi^n_{p,\mu} - \xi^n_{p',\mu}}{p - p'} \right| < \varepsilon.$$

This proves the claim. ■

Fix N large enough so that for every $n \geq N$, (32) holds and (31) is strictly increasing in μ in view of Lemma 3. Fix one such n. Because there is a positive measure of μ given which $w^n_{p,\mu}$, given in (9), is positive, the $\mathbf{E}^F[w^n_{p,\mu}]$ is strictly increasing in μ.

Let V^U_n denote the the uninformed type's payoff V^U_n in this equilibrium when the discount factor is δ_n. Then, for each $p \in [p_0, 1]$,

$$V^U_n(p) = \mathbf{E}^F \left[(1 - \delta) w^n_{p,\mu} + \delta \mu V^U_n \left(\frac{p}{p + (1 - p)\alpha^n_{p,\mu}} \right) \right],$$

where $w^n_{p,\mu}$ is given by (30). To prove that V^U satisfies Property 2, which will complete the proof, note that the payoff $V^U_n(p)$ is the unique fixed point of the contraction T^U_n that maps from the set of continuous and strictly increasing functions $f : [p_0, 1] \to \mathbf{R}_+$

to itself when n is sufficiently large:

$$T_n^U f(p) = \mathbf{E}^F\left[(1-\delta)w_{p,\mu}^n + \delta\mu f\left(\frac{p}{p+(1-p)\alpha_{p,\mu}^n}\right)\right]$$
$$= \mathbf{E}^F\left[(1-\delta)w_{p,\mu}^n + \delta\mu f(g_{p,\mu}^n)\right]$$

This follows as $\alpha_{p,\mu}$ is continuous in p, as shown above, and $\mathbf{E}^F[w_{p,\mu}^n]$ is strictly increasing in p; moreover, by Part 3 of Lemma 1, $\alpha_{p,\mu}^n$ is increasing in p for each μ, and that by Claim 2, $g_{p,\mu}^n$ is strictly increasing in p.

Finally, the proof that V_n^I is strictly increasing in p is analogous and is therefore omitted.

B.6.2 Part 1 and Part 2

Proposition 2 has shown that the condition $\mu \in [\underline{\mu}^*, \bar{\mu}^*]$ is equivalent to the condition $\alpha_{p,\mu}^* - \kappa_{p,\mu} \leq 0$. I apply Lemma 1 below to prove the two parts of this proposition. Let $\{\delta_n\}_{n=0}^{\infty}, \alpha^*$, and $\alpha^n : (0,1]$ be defined as in Section 4. Fix N such that for every $n \geq N$:

- (7) and (8) in Claim 1 hold,
- $\alpha_{p,\mu}^n$ is increasing in p in view of Part 3 of Lemma 2.

Fix some $n \geq N$.

Part 1. Because $\alpha_{p,\mu}^* - \kappa_{p,\mu}$ is strictly convex in μ on $[\frac{1}{2}, 1)$, by (8), $\alpha_{p,\mu}^n - \kappa_{p,\mu}$ is strictly convex in μ on $[\frac{1}{2}, 1)$. In turn, by (7), there exists $\bar{p}^n$ in the neighborhood of p^*, given in (11), such that if $p \leq \bar{p}^n$, then $\alpha_{p,\mu}^n - \kappa_{p,\mu}$ admits roots $\underline{x}^n(p), \bar{x}^n(p)$, satisfying:

1. $\underline{x}^n(p) \leq \bar{x}^n(p)$;
2. $\underline{x}^n(p)$ is in the neighborhood of $\underline{\mu}^*(p)$,
3. $\underline{x}^n(p)$ is in the neighborhood of $\bar{\mu}^*(p)$,
4. $\underline{x}^n(\bar{p}^n) = \bar{x}^n(\bar{p}^n)$.
5. $\alpha_{p,\mu}^n - \kappa_{p,\mu} \leq 0$ if and only if $\mu \in [\underline{x}^n(p), \bar{x}^n(p)]$.

If $p > \bar{p}^n$ instead, then $\alpha^n_{p,\mu} - \kappa_{p,\mu}$ admits no root. Because $\frac{1}{2} < \underline{\mu}^*(p) \leq \bar{\mu}^*(p) < 1$, by construction, $\frac{1}{2} < \underline{x}^n(p) \leq \bar{x}^n(p) < 1$. Setting

$$\underline{\mu}^n(p) := \begin{cases} \bar{x}^n(p), & \text{if } p < \bar{p}, \\ \bar{x}^n(\bar{p}), & \text{otherwise,} \end{cases}$$

$$\bar{\mu}^n(p) := \begin{cases} \underline{x}^n(p), & \text{if } p < \bar{p}, \\ \underline{x}^n(\bar{p}), & \text{otherwise,} \end{cases}$$

proves Part 1.

Part 2. Because $\alpha^n_{p,\mu}$ is increasing in p and $\kappa_{p,\mu}$ is strictly decreasing in p, the expression $\alpha^n_{p,\mu} - \kappa_{p,\mu}$ is strictly increasing in p. Thus, the two roots $\underline{x}^n(p)$ and $\bar{x}^n(p)$ identified in Part 1 given which $\alpha^n_{p,\mu} - \kappa_{p,\mu} = 0$ for each $p < \bar{p}^n$, must satisfy:

1. $\underline{x}^n(p)$ is strictly increasing in p;
2. $\underline{x}^n(p)$ is strictly decreasing in p.

By definition of $\underline{\mu}^n(p)$ and $\underline{\mu}^n(p)$, the claim in Part 2 follows.

B.7 Proof of Corollary 1

This claim follows from the following two observations in the equilibrium:

1. The expert's wage in any period t, given that no past report is incorrect and the public beliefs are (p_t, μ_t), is

$$w_t = \max\left[0, p_t + (1-p_t)(\alpha_{p_t,\mu_t}\mu_t + (1-\alpha_{p_t,\mu_t})(1-\mu_t)) - \mu_t\right],$$

   according to (5). This wage tends to $1-\mu_t$ as $p_t \to 1$.

2. Conditional on an informed type, all reports are correct on path and any realized path of the expert's reputation $(p_t)_{t=0}^{\infty}$ is weakly increasing. In addition, given any outcome of the game, for each period t, $p_t = p_{t+1}$ if and only if in period t, the true state is 0 and μ_t is sufficiently close to one so that the uninformed type reports state 0 with probability one. The probability that this latter event

happens in T consecutive periods tends to zero as T tends to infinity. Thus, for every $\varepsilon > 0$, there exists T such that for every $t \geq T$,

$$\mathbf{P}[w_t < 1 - \mu_t - \varepsilon | \theta = \theta_I] < \varepsilon.$$

B.8 Proof of Corollary 2

Suppose that F is degenerate at some $\breve{\mu} \in [\frac{1}{2}, 1)$. In the equilibrium, conditional on an informed type, the wage in each period t is

$$w_t = \max\left[0, p_t + (1 - p_t)(\alpha_{p_t,\breve{\mu}}\breve{\mu} + (1 - \alpha_{p_t,\breve{\mu}})(1 - \breve{\mu})) - \breve{\mu}\right].$$

By Part 3 of Lemma 2, when the discount factor is sufficiently close to one, $\alpha_{p_t,\breve{\mu}}$ is increasing in p_t. Thus, w_t is increasing in p_t.

B.9 Proof of Proposition 4

Suppose that the state is publicly realized with some probability $v \in [0, 1)$ and is hidden otherwise after the DM takes his action. The uninformed type's equilibrium strategy $\tilde{\alpha}^*_{p,\mu}$ is characterized by her incentive constraints, where she compares her posterior reputation by reporting 0 and that by reporting 1. Specifically, if

$$v\mu p + (1 - v)\left(\frac{p\mu}{p\mu + 1 - p}\right) \geq v(1 - \mu) + (1 - v), \tag{33}$$

then she reports state 0 for sure, i.e., $\tilde{\alpha}^*_{p,\mu} = 1$; otherwise, $\tilde{\alpha}^*_{p,\mu} < 1$ and it is characterized by the indifference condition

$$\begin{aligned} v\mu &\left(\frac{p}{p + (1 - p)\tilde{\alpha}^*_{p,\mu}}\right) + (1 - v)\left(\frac{p\mu}{p\mu + (1 - p)\tilde{\alpha}^*_{p,\mu}}\right) \\ &= v(1 - \mu)\left(\frac{p}{p + (1 - p)(1 - \tilde{\alpha}^*_{p,\mu})}\right) + (1 - v)\left(\frac{p(1 - \mu)}{p(1 - \mu) + (1 - p)(1 - \tilde{\alpha}^*_{p,\mu})}\right). \end{aligned} \tag{34}$$

This proposition concerns with the case that $v = 0$. In this case, (33) fails and by (34), $\tilde{\alpha}^*_{p,\mu} = \mu$. Anticipating this strategy, the DM optimally matches his action with a report of state 0 for the same reason as in Proposition 1. He optimally takes action

0 as well despite receiving report 1 if and only if $\tilde{\alpha}^*_{p,\mu} = \mu < \kappa_{p,\mu}$, or equivalently, $\mu > \hat{\mu}(p) := [2(1-p)]^{-1}$. Because $\hat{\mu}(p)$ is strictly increasing in p, the proof is complete.

B.10 Proof of Proposition 5

Consider (33). Observe that there exists $\underline{v}$ such that for every $v \geq \underline{v}$, there is $\underline{\mu}_v \in (\frac{1}{2}, 1)$ such that if $\mu \geq \underline{\mu}_v$, then (33) holds. Thus, for any such values (v, μ), $\tilde{\alpha}^*_{p,\mu} = 1$ in the equilibrium. Because $\kappa_{p,\mu} < 1$, there is no crisis of expertise.

B.11 Proof of Proposition 6

Fix μ and then fix $\lambda \in (\mu, 1)$. In this extension, the uninformed type's equilibrium strategy $\tilde{\alpha}^*_{p,\mu}$ is characterized by the indifference condition between reporting either state:[17]

$$\begin{aligned} &\mu\left(\frac{p\lambda}{p\lambda + (1-p)\tilde{\alpha}^*_{p,\mu}}\right) + (1-\mu)\left(\frac{p(1-\lambda)}{p(1-\lambda) + (1-p)\tilde{\alpha}^*_{p,\mu}}\right) \\ &\quad = (1-\mu)\left(\frac{p\lambda}{p\lambda + (1-p)(1-\tilde{\alpha}^*_{p,\mu})}\right) + \mu\left(\frac{p(1-\lambda)}{p(1-\lambda) + (1-p)(1-\tilde{\alpha}^*_{p,\mu})}\right). \end{aligned} \tag{35}$$

As $\mu \to 1$ and $\lambda \to 1$, $\tilde{\alpha}^*_{p,\mu} \to \alpha^*_{p,\mu} = 1$. The DM's cutoff $\kappa_{p,\mu}$ is unaffected by the informed type's signal precision and is given by (2). Because $\kappa_{p,\mu}$ is continuous and strictly increasing in μ, and $\lim_{\mu \uparrow 1} \kappa_{p,\mu} = 1$, there exists $\underline{\lambda} \in (0, 1)$ such that for every $\lambda \geq \underline{\lambda}$, there is $\bar{\mu}_\lambda \in (\frac{1}{2}, 1)$ such that if $\mu \geq \underline{\mu}_\lambda$, $\tilde{\alpha}^*_{p,\mu} \geq \kappa_{p,\mu}$. The claim thus follows.

B.12 Proof of Proposition 7

Suppose that $\mu' \geq \frac{1}{2}$. Following the proof of Proposition 1, given the DM's expectation $\alpha^*_{p,\mu}$ of the uninformed type's report probability, the DM matches his action with report 0 for sure; in contrast, he matches his action with report 1 if and only if

$$\alpha^*_{p,\mu} \geq \kappa_{p,\mu'} = \frac{\mu'(2-p) - 1}{(2\mu' - 1)(1-p)}.$$

[17] Different from (3), here, the uninformed type reports state 1 with positive probability irrespective of (p, μ) in equilibrium. This is because if the DM anticipates that she reports state 0 for sure and knows that the informed type's signal might not match the state, then the uninformed type profits by deviating to report state 1 and ensures a reputation of one.

Because $\kappa_{p,\mu'} \leq 1$, and $\alpha^*_{p,\mu} = 1$ for μ close to 1, the claim follows.

Suppose next that $\mu' < \frac{1}{2}$. Then, given the DM's expectation $\alpha^*_{p,\mu}$ of the uninformed type's report probability, the DM matches his action with report 0 if and only if

$$\frac{p\mu'}{p\mu' + (1-p)\alpha^*_{p,\mu}} + \frac{(1-p)\alpha^*_{p,\mu}}{p\mu' + (1-p)\alpha^*_{p,\mu}} \cdot \mu' \geq \frac{(1-p)\alpha^*_{p,\mu}}{p\mu + (1-p)\alpha^*_{p,\mu}}(1-\mu')$$

$$\iff \qquad \alpha^*_{p,\mu} \leq \frac{\mu' p}{(1-2\mu')(1-p)}.$$

Note that the right side of this inequality equals 0 at $\mu' = 0$, is strictly increasing in μ', and tends to infinity as μ' tends to infinity. Thus, given any (p, μ) that fix $\alpha^*_{p,\mu}$, the DM matches his action with report 0 if and only if μ' is sufficiently close to $\frac{1}{2}$. On the other hand, the DM optimally matches his action with report 1 if and only if

$$\frac{p(1-\mu)}{p(1-\mu) + (1-p)(1-\alpha^*_{p,\mu})} + \frac{(1-p)(1-\alpha^*_{p,\mu})}{p(1-\mu) + (1-p)(1-\alpha^*_{p,\mu})}(1-\mu)$$
$$> \frac{(1-p)(1-\alpha^*_{p,\mu})}{p(1-\mu) + (1-p)(1-\alpha^*_{p,\mu})} \cdot \mu.$$

This inequality always holds; thus, the DM matches his action with report 1 for sure.

B.13 Proof of Lemma 4

Suppose that the DM assigns a positive belief $p^o_1 > 0$ that the expert is informed when it anticipates the uninformed type to only report 0 but then receives a report 1 which turns out to be incorrect. Suppose further that the beliefs (p, μ) satisfy

$$p\mu < (1-\mu) + \mu p^o_1, \tag{36}$$

$$p\mu \geq 1 - \mu. \tag{37}$$

The inequality (36) ensures that if the DM anticipates the uninformed type to only report 0, then the uninformed type has a profitable deviation to report 1. Moreover, as Section 2, if the DM anticipates the uninformed type to only report 1, then she has a profitable deviation to report 0 irrespective of the DM's off-path belief that she is informed if the report 0 turns out to be incorrect. Thus, in equilibrium, the uninformed type must report both 0 and 1 with positive probabilities $\alpha_{p,\mu}$ and $1 - \alpha_{p,\mu}$. But then

(37), which states that the expert's prior reputation is sufficiently high, implies that no such $\alpha_{p,\mu} \in (0,1)$ exists. If there were one such $\alpha_{p,\mu}$, then the uninformed type's incentive constraint must hold:

$$\mu \frac{p}{p + (1-p)\alpha_{p,\mu}} = (1-\mu) \frac{p}{p + (1-p)(1-\alpha_{p,\mu})}. \tag{38}$$

Because $\alpha_{p,\mu} \in (0,1)$, the left side of (38) strictly exceeds $p\mu$ and the right side of (38) is strictly smaller than $1-\mu$, contradicting (37). In contrast, if (37) holds with the reverse inequality, then no contradiction arises and an equilibrium exists. The equilibrium probability $\alpha_{p,\mu}$ is then determined by (38) as in Section 2.

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